Interferometric Interpolation of 3D SSP Data

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Summary:

We present the theory and numerical results for interferometrically interpolating 3D marine surface seismic profiles (SSP) data. For the interpolation of SSP data we use the combination of a natural Green's function (SSP shot gathers) and a model-based Green's function for the water-layer model. Synthetic results show that the aliased SSP data with sparse receiver intervals can be accurately interpolated to smaller intervals. The virtual shot gather contains some artifacts so a non-stationary 2D multi-channel image matching filter is used after interferometric interpolation to remove these artifacts. Results suggest that a sparse marine SSP survey can yield more information about the reflectors if data are interpolated by interferometry. This assumes that the sources are located both outside and above the recording aperture.

Introduction:

Typical marine surface seismic profiles (SSP) are ideally designed for a regular recording grid, but in practice surveys suffer from irregularities in the recording geometry, coarse receiver spacing, and narrow recording apertures, especially in the crossline direction. The result can be inadequate subsurface illumination and distortions in the migration image.

To alleviate this problem, various algorithms were suggested to fill in the missing traces in marine data (Abma and Kabir, 2006; van Dedem and Verschuur, 2005; and Muijs et al., 2007). Most of them require certain assumptions such as, the data have a sparse representation in a certain domain, or rely on a priori information, such as knowledge of seismic velocities (Baumstein et al., 2005). In most cases, they do not use the redundant information available in the free-surface multiples.

Multiples are often considered as noise therfore are removed from data. This appears to ignore the benefit that multiples are second and third views of the subsurface that provide either redundant or new information about the subsurface.

Berkhout and Verschuur (2006), Ramirez et al. (2007), and Curry and Shan (2006) used multiples to interpolate missing traces in seismic survey. In Berkhout and Verschuur (2006), two steps are required to interpolate the missing traces: first, an inverse focal transformation is obtained to separate primaries from multiples and second, the separated multiples are transformed to primaries and used to fill in the gaps. Ramirez et al. (2007) presented a way to interpolate and extrapolate missing traces by interferometry, where direct waves and free-surface ghosts are crosscorrelated with measured field data to estimate the missing traces. Their method requires the presence of both the pressure wavefield and the normal derivative. If the normal derivative data are not available two approximations are made: (1) the medium at the surface is assumed to be homogeneous and (2) the local angle between the ray approximation of the wave field and the vector normal to the surface is assumed to be zero. In Curry and Shan (2006) both the primary-only data and the multiple-only data were used to generate virtual primaries. A disadvantage of this approach is that it requires the primary-only and multiple-only data, which is not always available.

In this paper we show how to interpolate marine SSP data by interferometry using a model-based Green's function. This method transforms surface and seabed related multiples into primaries recorded at virtual receivers both inside and outside the receiver array. In this proposed technique, no assumptions and no velocity model for the deeper sediments is needed. The interpolated data can be used for migration, velocity analysis, and tomography. The multiples can also be used to illuminate much wider areas of the subsurface. This work is the 3D extension of the 2D interferometric method of Dong and Hanafy (2008) and Dong et al. (2009).

Method:

Wang et al. (2009) applied the interferometric interpolation method to surface seismic profile (SSP) data. In that method, the surface related multiples are used to predict missing traces in gaps. Sparse SSP data can also be interpolated where an SSP → SSP correlation transform is used. This transform is similar in spirit to the VSP → VSP correlation transform (Schuster, 2009), except in our current work we now use a two-layer model-based Green's function with a band limited point source, as shown in Figure 1. The diagrams show how SSP traces (with both sources and receivers just below the free surface) are correlated with sparsely distributed SSP traces to generate a dense distribution of SSP traces. This correlation operation is required by the acoustic reciprocity equation of correlation type for a two-state system, where one state is the acoustic field associated with the multi-layered model shown in Figure 1 and the other state is associated with a sea-floor model. It also is desirable to use only the upgoing events, although this does not always appear to be a necessary requirement if a matching filter is used.

The reciprocity equation of correlation type can be described as:

$$G(\mathbf{B}|\mathbf{A}) - G_0(\mathbf{A}|\mathbf{B})^* = \int_{S_s} [G_0(\mathbf{x}|\mathbf{B})^* \frac{\partial G(\mathbf{x}|\mathbf{A})}{\partial n_x} - G(\mathbf{x}|\mathbf{A}) \frac{\partial G_0(\mathbf{x}|\mathbf{B})^*}{\partial n_x}] d^2x, \quad (1)$$

where, $G_0(\mathbf{x}|\mathbf{B})$ is the model-based Green's function for a water layer model and $G(\mathbf{x}|\mathbf{A})$ is the Green's function for the actual earth model (Dong and Hanafy, 2008). Here, S_s is the boundary just below the sea surface and the integration along the free surface vanishes because both Green's functions are zero there. The contributions from the vertical boundaries at infinity to the left and to the right of the boat will be ignored (Wapenaar and Fokkema, 2006). Here, **A** and **B** are just below the free surface and below the horizontal source line S_o

The above equation is a reciprocity equation of correlation type for two different states, which can be used for interpolation or extrapolation of traces. The far-field approximation to equation 2 yields the SSP → SSP far-field transform

$$\overbrace{G(\mathbf{B}|\mathbf{A})}^{SSP} = 2ik\int_{S_s} \overbrace{G(\mathbf{x}|\mathbf{A})}^{SSP}\,\overbrace{G_0(\mathbf{x}|\mathbf{B})^*}^{SSP}\,dx^2 + \overbrace{G_0(\mathbf{A}|\mathbf{B})^*}^{SSP}, \qquad (2)$$

where only upgoing waves are considered. To implement this equation, the SSP data are used to estimate the upgoing portion of $G(\mathbf{x}|\mathbf{A})$ and a finite-difference solution to the wave equation is used to estimate the upgoing portion of $G_0(\mathbf{x}|\mathbf{B})^*$ for the two-layer model that only contains the free surface and ocean bottom interfaces. This FD calculation is possible because the sea floor topography is well known beneath any exploration survey. The key idea for interpolation is that the free surface acts as a perfectly reflecting mirror so that 2^{nd} and 3^{rd} views, i.e., free-surface related multiples, of the subsurface can be used to fill in the trace gaps, as indicated by Figure 1.

Numerical Test:

The proposed approach is tested on several velocity models. In this work, only the results from the SEG/EAGE velocity model (Figure 2) are presented. We simulated a near azimuth marine survey with 12 streamers, where each one is composed of 310 receivers at 25 m intervals. The crossline streamer interval is 150 m, the near offset hydrophone is at 200 m, and the far offset hydrophone is at 7925 m. The generated CSG has 3001 samples/trace at a temporal sample interval of 4 ms, which gives a total time of 12 s. Our goal is to interpolate virtual traces in both the inline and the crossline directions, so that the final data set consists of traces for 34 streamers at a 50 m streamer interval and 619 receivers per streamer at a receiver interval of 12.5 m. Figure 3 shows the first two streamers of the input CSG, and Figure 4 shows the virtual CSG (1^{st} to 4^{th} streamers) after one iteration. Artifacts due to the interferometry algorithm are shown in the virtual CSG (Figure 4). To minimize these artifacts and improve the final results the following 5 steps are used:

1. A 2D multi-channel local image matching filter is used after interferometric interpolation to remove the generated artifacts.
2. This non-stationary matching filter used in step 1 is repeated several times (8 times in this work) to enhance the virtual CSG.
3. All virtual traces that are at the actual trace location are replaced with these true traces.
4. The output of step 3 is used as input to another interpolation process.
5. Finally, steps 1 - 4 are repeated several times (3 times in this work) to produce the final virtual CSG.

Figure 5 shows the final virtual CSG, where the artifacts are largely eliminated. Comparing the final virtual CSG (Figure 5) with a synthetically generated CSG (Figure 6) that has the same parameters as the virtual CSG shows a very good correlation. For a more accurate comparison, Figure 7 shows a trace comparison between the interferometrically and synthetically generated CSGs. Figure 7 shows that the artifacts at the virtual CSG are largely eliminated and the amplitude values are mostly corrected.

Conclusions:

An interferometric method for interpolating marine SSP data is tested on the synthetic traces generated from the 3D SEG/EAGE velocity model. The results show that this method can kinematically interpolate the sparse SSP data to a dense receiver distribution of traces. The least squares image matching filter is shown to suppress artifacts and can partly correct for amplitudes.

No exact velocity model is required for this approach; however, the thickness of the water layer and the velocity of the seismic waves in the water are required and a rough estimate of the sediment velocity is desired. An exact one is not needed because of correction associated with the matching filter. The interpolated data can be used for migration, velocity analysis, and tomography. The multiples can also be used to illuminate much wider areas of the subsurface.

A problem with this approach is that we do not know the quantitative limits of this approach and at this stage have not related interpolation error to trace spacing. The appendix is the first step in quantifying such errors.

Extrapolating receivers to the area between the shot and the first receiver might be feasible with the same proposed approach. It is similar to the interpolation approach, but our extrapolated traces (not included in this work) show stronger artifacts especially at the near offset zone. A future challenge is to improve the quality of the extrapolated traces and eliminate the artifacts.

Acknowledgments:

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APPENDIX A

ANTI-ALIASING THE INTERFEROMETRIC INTERPOLATION

For simplicity in exposition we assume that $G(\mathbf{x}|\mathbf{B})$ only contains primary and $G(\mathbf{x}|\mathbf{A})$ contains primary and first order ghost, the interferometric interpolation equation (equation 2) can be formulated as shown below:

$$G(\mathbf{B}|\mathbf{A}) \approx 2ik\sum_{i=1}^{N}\phi(\mathbf{A},\mathbf{B},\mathbf{x_i}) \approx Ae^{i\omega(\tau_{\mathbf{Ax}_i}-\tau_{\mathbf{Bx}_i})}, \qquad (\text{A-1})$$

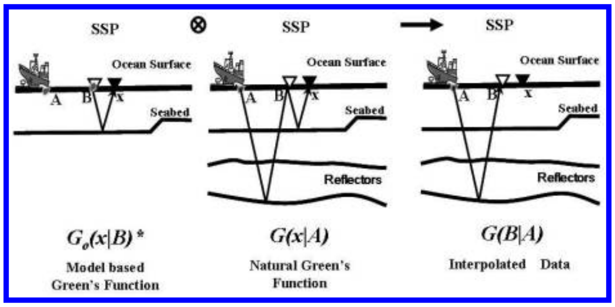

Figure 1: Ray diagrams for transforming SSP data to SSP data. Here, the open geophones indicate the locations of virtual geophones where traces are created from the original SSP data recorded at the filled geophone positions. Both $G(\mathbf{x}|\mathbf{A})$ and $G_o(\mathbf{x}|\mathbf{B})$ can be data based Green's functions, but in this case $G_o(\mathbf{x}|\mathbf{B})$ is computed for the two-layer sea-floor model.

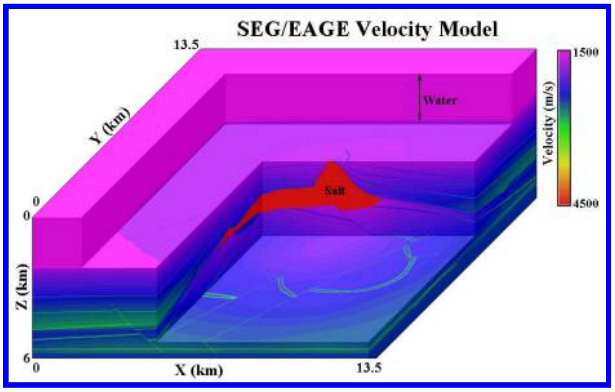

Figure 2: The SEG/EAGE velocity model used to test the interpolation technique.

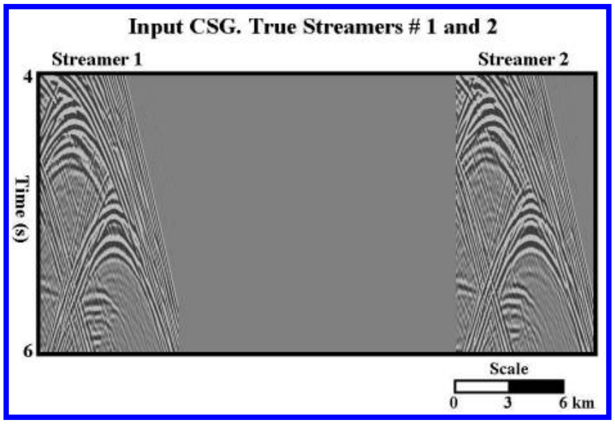

Figure 3: Traces associated with the 1^{st} and 2^{nd} streamers of the original common shot gather zoomed between 4-6 s for better display. Here the inline receiver interval is 25 m, the crossline streamer interval is 150 m, each streamer consists of 310 receivers, and the total number of receivers in the input data are 3,720.

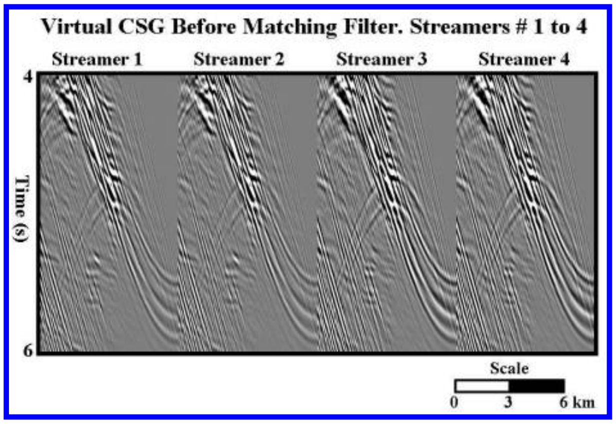

Figure 4: The interpolation results of the SEG/EAGE velocity model with one interpolation iteration and no matching filter, where there are many artifacts shown in this shot gather. This figure shows traces for streamers 1 to 4 and is zoomed between 4-6 s for better display.

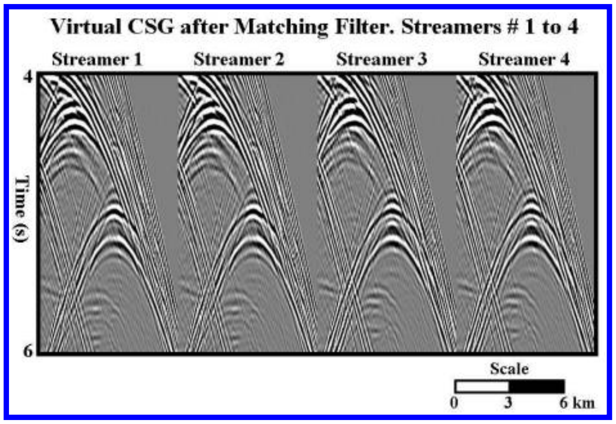

Figure 5: The interpolation results after using 3 iterations of interpolation after a sequence of matching filters is used to remove artifacts and enhance the data. Most of the artifacts are removed and the wavelet is consistent with the true data. The virtual CSGs consists of 34 crosslines with 619 receivers per crossline. The inline receiver interval is 12.5 m, the crossline spacing is 50 m, and total number of receivers in the output data are 21,046. This figure shows streamers 1 to 4 and zooms in between 4-6 s for better display.

where $\phi(\mathbf{A},\mathbf{B},\mathbf{x_i}) = G(\mathbf{x_i}|\mathbf{A})G_0(\mathbf{x_i}|\mathbf{B})^*$ denotes a correlated trace, $\mathbf{x}_i$ is the location of the i^{th} receiver, and N is the total number of SSP receivers. To avoid aliasing artifacts in the summation of these discretely sampled data $\phi(\mathbf{A},\mathbf{B},\mathbf{x_i})$, the phase difference between $\phi(\mathbf{A},\mathbf{B},\mathbf{x_i})$ and $\phi(\mathbf{A},\mathbf{B},\mathbf{x_{i+1}})$ needs to be less than π. Regarding the geometry shown in Figure A-1, we have the following relationship:

$$(\tau_{\mathbf{Ax}_{i+1}} - \tau_{\mathbf{Bx}_{i+1}}) - (\tau_{\mathbf{Ax}_i} - \tau_{\mathbf{Bx}_i}) < \frac{T}{2}, \tag{A-2}$$

where $\tau_{\mathbf{Ax}_{i+1}}$ and $\tau_{\mathbf{Ax}_i}$ are the multiple reflection traveltimes from the source $\mathbf{A}$ to the receivers $\mathbf{x}_{i+1}$ and $\mathbf{x}_i$ respectively, $\tau_{\mathbf{Bx}_{i+1}}$ and $\tau_{\mathbf{Bx}_i}$ are the primary reflection traveltimes from receiver locations $\mathbf{x}_{i+1}$ and $\mathbf{x}_i$, respectively, to the virtual receiver location $\mathbf{B}$, and T is the dominant period of the data.

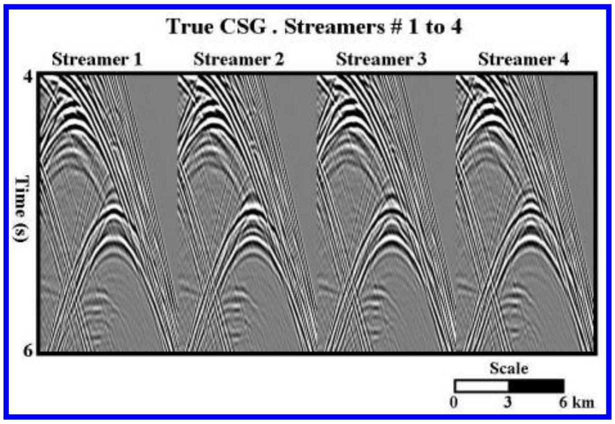

Figure 6: Synthetically generated CSG with the same geometry as in Figure 5. This Figure shows streamers 1 to 4 and zoomed between 4-6 s for better display.

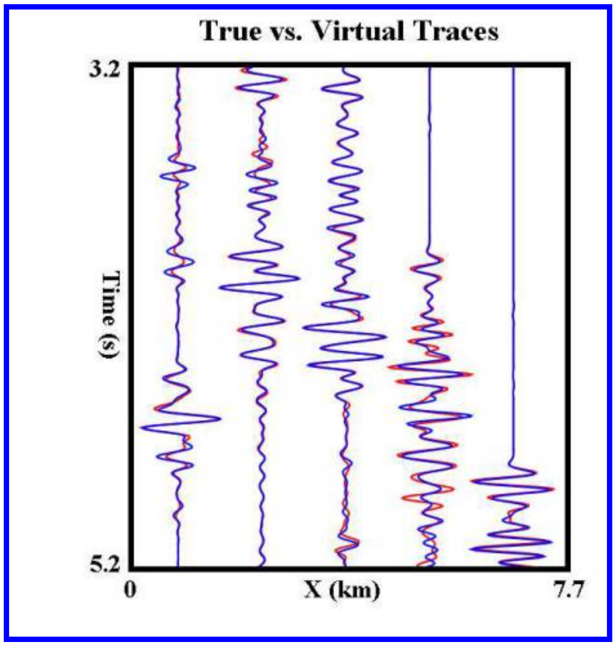

Figure 7: Trace comparisons between interferometrically generated (Figure 5) and actual (Figure 6) CSGs. These traces are extracted from streamer 2, where all receivers in this streamer are virtual. We zoomed between 3.2-5.2 s for better display, where the red lines represent actual traces and blue lines represent virtual traces.

Rearranging equation A-2 gives

$$(\tau_{\mathbf{Ax}_{i+1}} - \tau_{\mathbf{Ax}_i}) - (\tau_{\mathbf{Bx}_{i+1}} - \tau_{\mathbf{Bx}_i}) < \frac{T}{2}. \qquad \text{(A-3)}$$

If we make the plane wave assumption for both the multiple and primary reflection events, we have $\tau_{\mathbf{Ax}_{i+1}} - \tau_{\mathbf{Ax}_i} \approx \frac{\Delta x}{{v_x}^m(\mathbf{x}_i)}$, and $\tau_{\mathbf{Bx}_{i+1}} - \tau_{\mathbf{Bx}_i} \approx \frac{\Delta x}{{v_x}^p(\mathbf{x}_i)}$, where Δx is the recording interval, and ${v_x}^m(\mathbf{x}_i)$ and ${v_x}^p(\mathbf{x}_i)$ are respectively the horizontal apparent velocities for the multiple and primary events at receiver location $\mathbf{x}_i$. Then equation A-3 is written as

$$\Delta x\left(\frac{1}{{v_x}^m(\mathbf{x}_i)} - \frac{1}{{v_x}^p(\mathbf{x}_i)}\right) < \frac{T}{2}. \qquad \text{(A-4)}$$

Equation A-4 can be used as the anti-aliasing criteria for interferometric interpolation. Comparing equation A-4 to the anti-aliasing criterion for regular SSP acquisition $\Delta x \frac{1}{v_x(\mathbf{x}_i)} < \frac{T}{2}$ indicates that larger recording spacing is permitted for interferometric interpolation than for the regular SSP acquisition. This formula is easily generalized to the anti-aliasing criteria for the correlation of any pair of different events (e.g., 2^{nd}-order multiple correlated with a 1^{st}-order multiple).

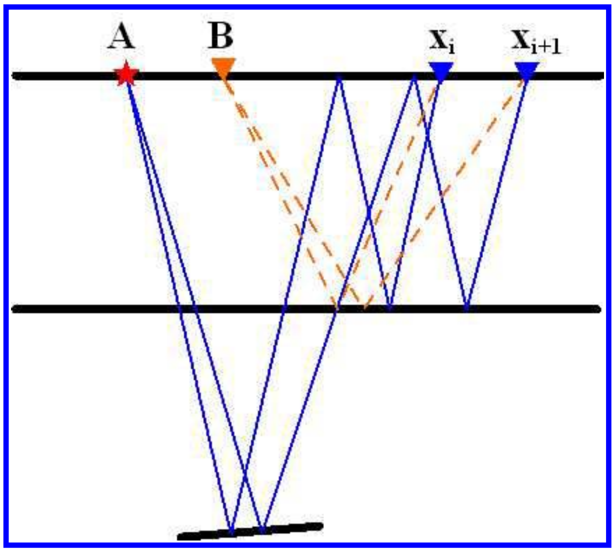

Figure A-1: SSP multiple reflections (blue rays) recorded at actual receivers (blue geophones) are correlated with the primary reflections (brown rays) to give a virtual trace at location **B**.

EDITED REFERENCES

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