Estimating the Water Reservoir Potential of a Wadi by Multiscale Early Arrival Waveform Inversion to Shallow Land Data

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ABSTRACT

We estimate the near-surface velocity distribution by applying multiscale early arrival waveform inversion (EWI) to shallow seismic land data. This data set is collected at Wadi Qudaid in western Saudi Arabia with the purpose of characterizing the shallow subsurface for its water storage and reuse potential. To enhance the accuracy of EWI, we corrected for the attenuation effects with an estimated factor Q, and also extracted a natural source wavelet from the data. We then applied EWI to invert the processed data for tomograms in different scales starting from a traveltime tomogram as our initial velocity model. Results suggest that, compared to traveltime tomography, EWI can generate a more highly resolved velocity tomogram from shallow seismic data by utilizing its low frequency components on coarse grids and its high frequency components on fine grids. The estimated water table in EWI tomogram is generally consistent with, but 9% deeper, than in the traveltime tomogram, showing that the water storage in this wadi might be less than expected from the traveltime tomogram. We believe that the more accurate EWI tomogram will make an economically important difference in assessing the storage potential of this wadi and wadis throughout the world.

INTRODUCTION

Traveltime tomography (Zhu and McMechan, 1989; Pratt and Goulty, 1991; Aki and Richards, 2002) is a ray-based geophysical imaging method that inverts the first-arrival traveltimes for the subsurface velocity distribution. It iteratively updates the velocity models by smearing the traveltimes residuals along the calculated ray paths (Nolet, 1987), and provides a smooth estimate of the earth’s velocity distribution. However, traveltime tomography employs a high-frequency assumption that conflicts with the finite-frequency bandwidth of seismic data, and so generates tomograms with low-to-intermediate resolution.

Full waveform inversion (FWI) (Tarantola, 1984) can overcome the high-frequency limitation in either the space-frequency (Pratt et al., 1998) or the space-time domains (Tarantola, 1986; Mora, 1987; Zhou et al., 1995). However, FWI is computationally expensive and its misfit function is highly nonlinear with respect to velocity perturbations. Therefore, Sheng et al. (2006) proposed an early arrival waveform tomography method in the time-space domain for near-surface refraction data. This approach can handle field data if we have a good approximation of the subsurface velocity (Buddenseik, 2004), and largely avoids having to account for dominant elastic effects in the data such as surface waves and P-to-S converted reflections. In this method, fewer events need to be fitted than in full waveform inversion, but local minima problems usually appear even if a time window is applied to seismic data. This phenomenon is attributed to high- frequency data used in the inversion, which causes the misfit function to be highly nonlinear.

In contrast, multiscale waveform inversion (MWI), proposed by Bunks et al. (1995), initially inverts the low-pass filtered data and then gradually incorporates the higher frequencies into the inversion. MWI is more likely to reach global minimum (Sirgue and Pratt, 2004) because the misfit function at low frequencies is more linear with respect to slowness perturbations than at high frequencies and cycle skipping problems are mitigated. Once a smooth or low-wavenumber tomogram is inverted from data at low frequencies, a higher wavenumber tomogram can be progressively reconstructed from the data at higher frequencies. A strategy to select optimal frequency bands for different scales was proposed by Sirgue and Pratt (2004).

In this work, our goal is to apply the multiscale early arrival waveform inversion (EWI) with an attenuation factor to a land data set collected at Wadi Qudaid, 100 km north of Jeddah, Saudi Arabia. Wadis are closely associated with centers
of human population, because subsurface water is often found and can be extracted by wells. As a result, we tend to locate the water table in that wadi area by applying EWI. Essential to locating the subsurface water is the precise characterization of the subsurface properties of the wadi. Such properties include the extent of the porous rock in the subsurface, the depth of the impermeable bedrock, and the presence of any faults that might drain the water from the storage reservoir. We propose to characterize the subsurface geology of a wadi by seismic MEWI, a method with much better resolution than traditional traveltime tomography (Bunks et al., 1995; Zhou et al., 1995, 1997; Sirgue and Pratt, 2004). Since the subsurface soils are partially to fully saturated, the attenuation factor $Q$ is required to correct for attenuation effects especially in the data at high frequencies before we invert for the velocity model. The waveform velocity tomogram is used to delineate the depth to the bedrock and the extent of the overlying porous rock, and therefore estimate the wadi’s potential for water storage. A local well log is used to identify porous rock from the seismic velocities in the tomogram.

In this paper, we first present the theory of multiscale EWI. In the following section, we introduce our seismic survey and data processing steps for the wadi data. The multiscale EWI and traveltime tomography methods are applied to the early arrivals of the 2D land data set and the resulting tomograms are compared to one another in the numerical results section. We also conduct a sanity test based on the final multiscale EWI tomogram to examine the early arrival waveform inversion method. The last section summarizes the salient results of our research.

**THEORY**

**Acoustic waveform inversion**

Early arrival waveform inversion (EWI) assumes the constant-density acoustic wave equation,

\[
\frac{1}{c^2(x)} \frac{\partial^2 p(x, t|x_s)}{\partial t^2} - \nabla \cdot p(x, t|x_s) = s(x, t|x_s),
\]

where $p(x, t|x_s)$ denotes the pressure field at position $x$, time $t$, and a source at $x_s$. The velocity model is represented by $c(x)$, and $s(x, t|x_s)$ is the source function. The solution to equation 1 is calculated by a finite-difference method (Levander, 1988) and can be written in terms of its Green’s function $g(x, t|x’, 0)$ as

\[
p(x, t|x_s) = \int g(x, t|x’, 0) \ast s(x’, t|x_s) dx’,
\]

where the symbol $\ast$ denotes temporal convolution. We can ignore the surface wave and shear effects in the elastic wave equation (Zhou et al., 1995), because the early arrivals in the 2D near-surface seismic data contain few elastic effects.

EWI estimates the velocity model by minimizing the early arrival misfit function (Sheng et al., 2006; Boonyasirinwat et al., 2010), where the waveform data residual is defined as

\[
\Delta p(x_s, t|x_s) = [p_{obs}(x_s, t|x_s) - p_{calc}(x_s, t|x_s)] W(x_s, t|x_s).
\]

Here, $x_s$ is the receiver position vector, $p_{obs}$ and $p_{calc}$ are, respectively, the observed and calculated pressure traces, and $W(x_s, t|x_s)$ is a window function that mutes all the energy except for the early arrivals. The velocity model $c(x)$ is iteratively updated by finding the slowness model that minimizes the misfit functional $E$, represented by the $L_2$ norm of the data residuals over time and space,

\[
E = \frac{1}{2} \sum_s \sum_g \int (\Delta p(x_s, t|x_s))^2 dt.
\]

A nonlinear conjugate-gradient method (Luo and Schuster, 1991) is used to minimize the gradient function. The gradient of the misfit functional $E$ with respect to changes in the velocity $c(x)$ is the first variation (Logan, 1996) of $E$ at the vector point $c(x)$ in the direction of $\delta c(x)$. This gradient $[\text{grad}(x) = \frac{\partial E}{\partial c(x)}]$ is computed by migrating the waveform residuals (Tarantola, 1984) with the following formula,

\[
\text{grad}(x) = \frac{1}{c^2(x)} \sum_s \sum_g \int \hat{p}_f(x, t|x_s) \hat{p}_b(x, t|x_s) dt;
\]

where $\hat{p}$ is the time derivative of $p$, $\hat{p}_f(x, t|x_s)$ represents the forward-propagated wavefields, and $\hat{p}_b(x, t|x_s)$ represents the back-projected waveform residual wavefields given by

\[
\hat{p}_b(x, t|x_s) = \int g(x, -t|x’, 0) \ast \delta p(x’, t|x_s) dx’,
\]

and

\[
\delta p(x’, t|x_s) = \int \delta p(x’ - x_s) \Delta p(x_s, t|x_s).
\]

Now the velocity model can be updated iteratively along the conjugate directions defined by

\[
d_k = -P_k g_k + \beta_k d_{k-1},
\]

for iterations $k = 1, 2, ..., k_{max}$. $g = [\text{grad}(x)]$, and $P$ is the conventional geometrical-spreading preconditioner (Causse et al., 1999). For the first iteration, we set $d_0 = -g_0$. The parameter $\beta_k$ is calculated by the Polak-Ribiére formula (Nocedal and Wright, 1999)

\[
\beta_k = \frac{g_k^T (P_k g_k - P_{k-1} g_{k-1})}{g_{k-1}^T P_{k-1} g_{k-1}},
\]

and the velocity model is updated by

\[
c_{k+1}(x) = c_k(x) + \lambda_k d_k(x),
\]

where $\lambda_k$ is the step length which can be determined by a quadratic line-search method (Nocedal and Wright, 1999), and $d_k(x)$ is the component of the direction vector $d(x)$ indexed by $x$. To compute the gradient direction at each iteration reduces to computing the reverse time migration operation. Additional forward modelings are required for the line search. The initial velocity model $c_0(x)$ is the traveltime tomogram inverted by picked first arrivals Nemeth et al. (1997), and equation 10 is iteratively applied until the misfit functional $E$ satisfies a stopping criterion.
Optimal frequency bands for EWI

Since multiscale EWI in the space-time domain uses a band of frequencies at each scale, it can update a much wider range of wavenumbers than in the frequency domain (Buddensieck, 2004). The formula to select upscaling frequencies proposed by Sirgue and Pratt (2004) in the frequency domain is

$$f_{n+1} = \frac{f_n}{\alpha_{\min}},$$  \hspace{1cm} (11)

where \(f_n\) is the current frequency, \(f_{n+1}\) is the next frequency to be chosen, and \(\alpha_{\min} = z/\sqrt{h^2 + z^2}\) is the parameter as a function of the maximum half-offset \(h\) and the maximum depth \(z\) in the image. The vertical wavenumber range \([k_{z \min}(f_n), k_{z \max}(f_n)]\) is calculated by

$$k_{z \min}(f_n) = \frac{4 \pi f_n \alpha_{\min}}{c_0},$$  \hspace{1cm} (12)

$$k_{z \max}(f_n) = \frac{4 \pi f_n}{c_0},$$  \hspace{1cm} (13)

where \(c_0\) is the smooth background velocity model. Equation 11 guarantees that the the lowest wavenumber to be updated at the \((n+1)\)-th frequency \(f_{n+1}\) is equal to the highest wavenumber at the \(n\)-th frequency \(f_n\). By 12 and 13, which are incorporated into 11, we obtain

$$k_{z \min}(f_{n+1}) = k_{z \max}(f_n).$$  \hspace{1cm} (14)

In our work, we inherit this frequency selection strategy in the time domain to reduce the overlapped regions of recovered wavenumber components in two neighboring scales. In time domain waveform inversion, a number of frequencies are used simultaneously for a given bandwidth. Here, we utilize equation 11 to choose \(f_n\) as the peak frequency for the frequency band in the \(n\)-th scale. Therefore, low-pass or bandpass filters with proper pass band and stop band windows can be applied to seismic data in different scales to control the recovered wavenumber range.

For a given frequency band, a grid size can be determined by the numerical stability and dispersion conditions associated with the finite-difference method at that scale. In this paper, the numerical dispersion condition for the finite-difference scheme is satisfied by more than ten grid points per dominant minimum wavelength (Levander, 1988). A square grid with \(dx = dz\) is applied. Once the space grid size is fixed, the time step \(dt\) can be determined by the 2D numerical stability condition (Courant et al., 1928):

$$dt < \frac{dz}{\sqrt{2c_{\max}}},$$  \hspace{1cm} (15)

where \(c_{\max}\) is the maximum velocity.

ACQUISITION AND PROCESSING

Near-surface refraction survey at Wadi Qudaid

The 2D seismic survey is conducted at Wadi Qudaid (see Figure 1) to the east of the campus of King Abdullah University of Science and Technology (KAUST). The 2D acquisition geometry (Figure 1b) consists of one line of vertical component geophones. Along this line, there are 117 receivers with a 2.0 m spacing, and the shots are located at every receiver position so \(117 \times 117 = 13689\) traces are recorded. In this field experiment, we used a 200 lb weight drop (Figure 2) to generate the seismic source energy with 10 ~ 15 stacks at each shot location. Each common shot gather (CSG) was recorded for 1 s with a sampling interval of 0.125 ms. Since we only need the early arrivals, arrivals after 0.25 s are muted. The CSG #30 and its picked first-arrival traveltimes are shown in Figure 3.

For this data set, we estimate the dominant wavelength and the dominant frequency of the first arrival head waves to be 6 m and 65 Hz, respectively, where the minimum P-wave velocity is estimated to be 380 m/s. Figure 4 shows the picked first arrival times for all traces presented as a 2D matrix, and Figure 5 shows the early arrival spectra of near-offset traces extracted from CSG #30 to CSG #50. The land data are processed before applying EWI in order to reduce elastic effects in the field data. The data processing steps include accounting for 3D wave propagation effects, attenuation effects, trace normal-
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tailed processing procedures.

Processing

1. The 3D land data are approximately transformed to 2D by multiplying the trace spectrum by $\sqrt{f/\omega}$ in the frequency domain and by multiplying the trace by $\sqrt{t}$ in the time domain (Buddenseik, 2004).

2. The attenuation effects in the field data should be corrected for because the forward modeling is based on the acoustic wave equation. According to Liao and McMechan (1997), the linear attenuation transfer function $T$ is a function of $f$, $t$, and $Q$, such that

$$T(f) = e^{-\left(\frac{\pi ft}{Q}\right)}$$

which transforms the input first arrival signal spectrum $S(f)$ to the output trace spectrum $R(f)$ by $R(f) = T(f)S(f)$. In equation 16, $f$ represents the frequency corresponding to the horizontal axis of the data spectrum, and $t$ is the first-arrival traveltime. Divided by the factor $T$, the attenuation in the recorded data spectrum $R(f)$ can be compensated and transformed into $S(f)$ using equation 16. In this study, $Q$ is fixed for the early arrivals, and is also assumed independent of $f$. The factor $Q$ can also be written in terms of the centroid frequencies $f_r$ and $f_s$ of the first arrival, respectively, at the receiver and the source as

$$f_r = f_s - \frac{2\pi \sigma_s^2}{Q} t,$$

where $\sigma_s^2$ represents the variance of the source spectrum. After $f_r$, $f_s$ and $\sigma_s^2$ are estimated from the data and the first arrivals $t$ are picked, the $Q$ value can be estimated from equation 17. Then, equation 16 is applied to the traces to correct the attenuation effects in the frequency domain. In this work, we estimate $Q$ as a constant because we only deal with early arrivals coming from a shallow part of the earth. Thus the same attenuation correction formula is used for all the traces.

3. All traces are normalized to reduce the errors resulting from previous correction steps. To retain only the early arrivals, the traces are also muted with a time window that starts about 3 to 4 periods after the first arrival. In this case, surface waves and later arrivals are not inverted by EWI.

4. To enhance the EWI results, a natural source wavelet is extracted by averaging the near-offset first arrivals within a window of 10 to 20 traces. We apply zero-lag cross correlation to several consecutive traces to align them to their first arrivals. Figure 6b shows the calculated natural source wavelet using the rectangular window (Figure 6a) to catch the first arrival wavelets for CSG #25 after the attenuation correction. Since the weight drop is controlled electronically, all shots have similar source wavelets as illustrated by Figure 7.
Figure 6: (a) The processed (steps 1∼5) CSG #25 after the application of a muting window and (b) the source wavelet extracted from the early arrivals.

Figure 7: Common offset gather with an offset equal to 200 m.

5. The very near-offset traces with source-receiver offsets no greater than 14 m are muted because they contain surface waves and noise even after filtering, and it is difficult to match them with the calculated seismograms. Noise before the picked first arrivals is also muted.

6. In the recorded wadi data set whose average peak frequency of the early arrivals is estimated to be 65 Hz, the initial filtered data for EWI have a peak frequency of $f_1 = 18$ Hz because most noise and surface waves are under 10 Hz in this data set. In our field experiment, we estimate the depth $z = 70$ m and the maximum half offset as $h = 116$ m, which leads to $f_2 = 0.52$ by $\alpha_{min} = z / \sqrt{h^2 + z^2}$. According to equation 11, we set $f_2 = 34$ Hz and $f_3 = 65$ Hz as the peak frequencies for the next two stage of two stages of EWI. Therefore, three bandpass filters with sequentially growing pass bands 0 ∼ 25 Hz, 0 ∼ 45 Hz, and 0 ∼ 75 Hz are respectively applied to all the traces in three different scales. These filters are also applied to the extracted source wavelet in step 4 such that EWI can be conducted at each scale with an appropriate source.

**NUMERICAL TESTS**

**EWI velocity inversion**

Figure 8: Pairs of the centroid frequencies and the first arrival traveltimes at the receivers. The attenuation factor $Q$ is estimated to be 28 by the best-fit line denoted by the solid yellow line.

Figure 8 shows the centroid frequencies $f_r$ plotted against traveltimes of the first arrivals. The variance $\sigma_s^2$ of the source centroid spectra is equal to 312.70 Hz$^2$, which is the average $\sigma_s^2$ value from all the sources (Liao and McMechan, 1997). Here, since the dominant peak frequency of the collected data is 65 Hz, we use a subband of 25 ∼ 90 Hz to calculate $f_r$, $f_s$, and $\sigma_s^2$ to avoid errors from the noise. By equation 17, the $Q$ value is estimated to be about 28, which is a typical value for near-surface soil with significant absorption. According to equation 16, $1/T(f)$ is applied to the trace spectra to correct for the attenuation effects in the frequency domain. Figure 9 shows CSG #117 before and after the attenuation correction, and Figure 10 demonstrates the spectrum comparisons of one trace before and after this operation. Figure 11 presents the raw and corrected CSG #26 before and after all the processing steps for three different scales. Figure 12 depicts three filtered source wavelets based on the extracted wavelet (see Figure 6b) for multiscalar EWI.

Figure 13a exhibits the smoothed traveltime tomogram which is used as the initial velocity model for EWI in the coarsest scale. Similarly, the tomogram inverted in the $n$-th scale is used as the initial velocity model in the $(n + 1)$-th scale. We set the spatial grid size $dx = 4$ m, 2 m, and 1 m for the coarsest, intermediate, and finest scales, respectively. The sampling interval $dt$ is equal to 0.125 ms and the total number of time
steps is 2000. In the coarsest scale, we use 58 CSGs (all the even numbered CSGs) instead of 117 CSGs to accelerate the computation by matching fewer data while all the CSGs are used in other two upcales. When conducting multiscale EWI, for the coarsest scale, we choose $p_f$ and $p_b$ instead of $\dot{p}_f$ and $\dot{p}_b$ in equation 5 to calculate the gradient by utilizing the low-frequency data. In the intermediate scale, we take the time derivative on either $p_f$ or $p_b$. Then we gradually turn to using $\dot{p}_f$ and $\dot{p}_b$ in the finest scale to attach more importance to high-frequency components in the data.

Figure 13b, 13c, and 13d show the inverted tomograms by EWI at different scales. Compared to the traveltime tomogram, multiscale EWI iteratively generates tomograms with gradually increasing resolutions by sequentially matching the collected data from low to high frequencies. In Figure 14, the processed CSG #116 are compared with the calculated data by EWI at each scale. Many early arriving events in the synthetic data correlate well with the observed traces in Figures 14a, 14b, and 14c. From the tomogram at the finest scale, we also estimate a water table curve according to the velocity contour of 1550 m/s. Relying on this curve, the estimated water table in the multiscale EWI tomogram is generally consistent with but 9% deeper than in the traveltime tomogram, showing that the...
water storage in this wadi might be less than expected from the traveltime tomogram. A well log reference of geological conditions above the 18 m deep water table is shown in Figure 15. The first layer (Figure 15a) consists of lose sand with gravels and the second layer (Figure 15b) consists of compact sand with some gravel and partially to fully saturated with water.

**Sanity Test on synthetic data**

In order to illustrate the effectiveness of EWI, we also conduct synthetic tests. The synthetic data set is generated based on the final velocity tomogram inverted by EWI in the finest scale, as shown in Figure 16a. The first arrival traveltimes of this synthetic data set are automatically picked and inverted by the traveltime tomography. The resulting traveltime tomogram is presented in Figure 16b. Compared to the original traveltime tomogram in Figure 13a, they match well in the 2D subsurface space. It is not necessary to carry out attenuation corrections for this synthetic data set since it is computed by a finite-difference solution to the acoustic wave equation. We then apply EWI using this new traveltimes tomogram as the initial velocity model by keeping the same size early arrival window as in the multiscale EWI. Figure 16c shows the tomogram inverted by EWI after 30 iterations. It shows highly resolved subsurface structures compared to the traveltime tomogram and validates EWI’s capability for improved resolution compared to traveltime tomography.

**CONCLUSIONS**

The multiscale early arrival waveform inversion method is used to invert seismic data collected at Wadi Qudaid. Compared to traveltime tomography, EWI does not require a high-frequency assumption and benefits from the attenuation correction applied to the recorded traces. EWI can be successfully applied to near-surface seismic data if careful processing steps are carried out before inversion. These steps include a low-pass or band-pass filter and corrections for 3D geometric spreading and attenuation effects, trace normalization, and accurate estimation of the source wavelet. Our synthetic and field data results suggest that EWI can create a more accurate and highly resolved velocity model compared to the traveltime tomogram. For reflections events from deep reservoir geology in oil industry, it is necessary to dynamically widen the early arrival window or inverting the deep reflections. Recall, it is important to recognize that EWI is designed to work well if the diving waves.
are in the early arrival window, and so the tomogram depth is limited to the diving wave’s depth of penetration. This is the main reason that EWI can be applied to inverting for shallow subsurface structures of the earth. Otherwise, it is necessary to dynamically open the arrival window to invert for the deeper reflections.

The drawbacks of EWI compared to traveltime tomography include complicated data processing steps and higher computational cost. Moreover, EWI fits complex waveforms instead of arrival times, which can be prone to cycle skipping and getting stuck in a local minimum, although low frequency data are first used in the inversion.

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