Preconditioned Least-squares Wave Equation AVP Migration

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Abstract

This paper presents a preconditioning implementation of regularized 3-D least-squares wave equation amplitude versus ray parameter (AVP) migration. The 3-D common azimuth migration/inversion is cast in a linear matrix formulation and solved by the conjugate gradients method. We study the implementation of pre-conditioning strategies to boost the convergence of the conjugate gradient algorithm. We process field data from the Western Canadian Basin using regularized least squares migration (LSM) and preconditioned LSM. A comparison of both stacked images and common image ray-parameter gathers shows that the preconditioned LSM can substantially decrease the computational cost of least squares migration without decreasing the quality of the inverted images.

Introduction

Regularized least-squares migration can provide high quality common image gathers (CIGs) even when the data are sparsely sampled (Kuehl and Sacchi, 2003; Wang, J., Kuehl, H. and Sacchi, M. D., 2003). A high computational cost makes least-squares methods impractical at first sight. An improvement in the convergence of the algorithm is needed to take full advantage of the technique.

Pre-conditioning strategies for semi-iterative solvers have been well studied by the applied mathematics community (Saad, Y., 1991; Hanke and Hansen, 1993). In addition, the problem has caught the attention of the geophysical community for interpolation (Fomel and Clearbout, 2003), Radon processing (Trad et al., 2003) and wave equation least-squares migration (Prucha and Biondi, 2002).

Regularized least-squares migration and preconditioned least-squares migration

In least-squares migration (Nemeth et. al, 1999) we express the migration problem as the solution of the following linear system:

$$d = Lm + n \ . \qquad (1)$$

Where d denotes the pre-processed data, L is the forward operator synthesized with the double-square-root operator and a local Radon transform to estimate ray-parameter gathers in depth, m is the common image gather parametrized as a function of x and y midpoint positions plus ray parameter along the x-offset direction, n denotes additive noise and errors arising from the fact that we approximate the true earth response by a simple linear operator L. Conventional migration applies the adjoint of L, denoted as L', to the data d. In general, we can consider migration as a low-resolution solution to the problem of inverting equation (1) (Nemeth et. al, 1999).

Equation (1) is inverted by solving a regularized inverse problem. Regularization is needed to obtain a unique and stable solution. In addition, regularization is used to impose a priori features to the image estimated via inversion. In this paper, the regularized solution is obtained by minimizing the following cost function:

$$F(m) = \| W(d - Lm) \|^2 + \lambda^2 \| Dm \|^2 \qquad (2)$$

where W is a diagonal weighting matrix used to penalize bad observations (missing observations). The operator D is a first order derivative operator along the in-line offset-ray-parameter direction. The trade-off parameter λ decides the amount of smoothing. The larger is λ, the smoother is the solution. The analytical solution of (2) is:

$$m = (L'W'WL + \mu D'D)^{-1} L'W'Wd \qquad (3)$$

The matrix $(L'W'WL + \mu D'D)$ cannot be inverted by direct solvers. Therefore, we use the conjugate gradients algorithm to minimize the cost function (3) without making any attempt to construct the system of normal equations.

The pre-conditioned solution is obtained by doing a simple change of variable

$$z = Dm \tag{4}$$

The substitution of m in equation (2) leads to

$$F(z) = \| W(d - LPz) \|^2 + \lambda^2 \| z \|^2$$
$$F(z) = \| W(d - \overline{L}z) \|^2 + \lambda^2 \| z \|^2 \tag{5}$$

where P should be the inverse of D. It is clear that rather than inverting D we will replace P by an operator with features similar to those of the inverse of D. If D is a discrete derivative operator; we can think of it as a high-pass operator or filter. Therefore, P must be an operator with a low-pass response. This rationale allows us to choose P as a low-pass convolution operator. In our implementation, to apply P is equivalent to apply 1-D convolution to common image gathers with a Hamming window. The convolution, in this context, is used to remove CIG artifacts arising from incomplete sampling, additive noise in the original data and errors in the operator (Kuehl and Sacchi, 2003).

Mathematically, the logic behind the preconditioning is that a good operator preconditioner will change the eigenvalue distribution of the operator (Saad, 1991). With the proper preconditioning, large eigenvalues will cluster together. Consequently, the CG method will require less iterations to minimize the cost function F given by equation (5).

Ray parameter sampling

The ray parameter axis should be sampled properly to avoid aliasing. Following Kostov (1990), we have adopted the following sampling criterion (Nyquist condition for slant stacks):

$$\Delta p_h \leq \frac{\Delta K_h}{\omega_{\max}} = \frac{2\pi / (N_h \cdot \Delta h)}{2\pi \cdot f_{\max}} = \frac{1}{N_h \cdot \Delta h \cdot f_{\max}} \tag{6}$$

where N_h is the number of offsets, Δh is the offset spacing and $f_{\max}$ is the maximum temporal frequency. We noticed that improper sampling often counterweights the benefits of pre-conditioning.

Field data example

We tested regularized LSM and preconditioned LSM using the Erskine data set provide by Veritas Geoservices. The data were binned to a common azimuth geometry. The data consist of 157 in-lines and 40 cross-lines. The offset ranges from zero to 3000 meters with a highly uneven distribution of offsets. The CMP gathers are quite sparse due to binning (Figure 1). Common image gathers (Figure 2) were extracted for the position: cross-line #10, in-line #71.

In Figures 2 and 3 we compare migrated versus inverted images. In the inversion case, we notice that by preconditioning we gain in computational efficiency. The regularized CG algorithm required 11 iterations to obtain the results portrayed in Figures 2C and 3B. On the other hand, only 4 iterations were utilized to achieve similar results when the pre-conditioning is applied (Figures 2D and 3C).

Summary

Least-squares AVP migration for common azimuth data has potential for deriving high resolution artifact-free CIGs that can be subsequently used to extract rock and/or fluid properties. It provides high quality common image gathers in ray parameter domain, and it also can increase the vertical resolution of stacked image. Preconditioned LSM can solve the inverse problem significantly more efficiently than regularized LSM.

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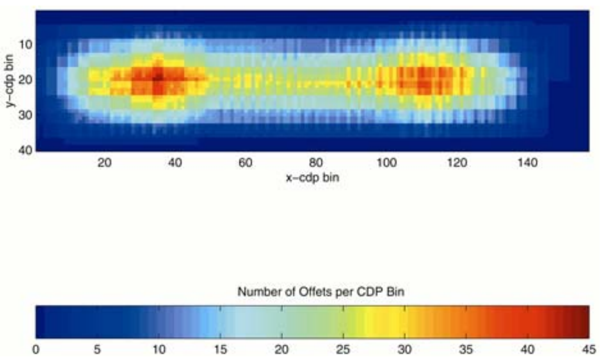

Figure 1 Distribution of offsets per CMP bin.

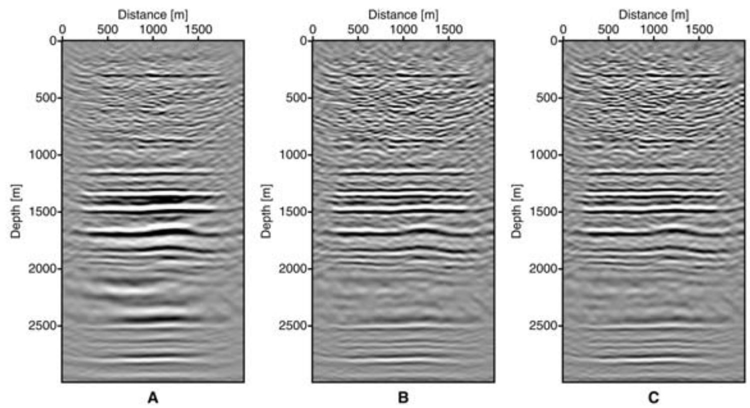

Figure 3 Stacked images for in-line #71. A) Migration. B) Regularized least-squares migration after 11 CG iterations. C) Preconditioned least-squares migration after 4 CG iterations.

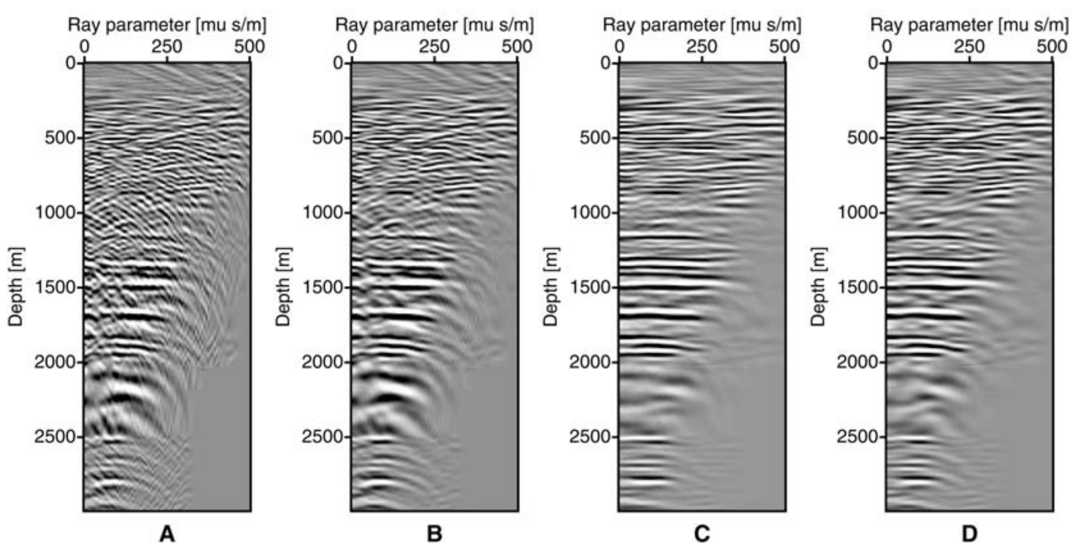

Figure 2 Common image gathers for the cross-line #10-in-line #71 position. A) Migration result. B) Regularized LSM after 4 CG iterations. C) Regularized LSM after 11 CG iterations D) Preconditioned LSM after 4 CG iterations.