1D Source Inversion

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1D acoustic wave equation is given by

$$\frac{1}{c^2(z)}\frac{\partial^2}{\partial t^2}p(z,t;z_s)-\frac{\partial^2}{\partial z^2}p(z,t;z_s)=\delta(z-z_s)s(t). \qquad (1)$$

Perturbing the source function $s(t)$ results in a perturbation of the pressure field $p(z,t;z_s)$ and equation (1) becomes

$$\frac{1}{c^2(z)}\frac{\partial^2}{\partial t^2}\left[p(z,t;z_s)+\delta p(z,t;z_s)\right]-\frac{\partial^2}{\partial z^2}\left[p(z,t;z_s)+\delta p(z,t;z_s)\right]=\delta(z-z_s)\left[s(t)+\delta s(t)\right]. \qquad (2)$$

Subtracting equation 1 from equation 2 yields

$$\frac{1}{c^2(z)}\frac{\partial^2}{\partial t^2}\delta p(z,t;z_s)-\frac{\partial^2}{\partial z^2}\delta p(z,t;z_s)=\delta(z-z_s)\delta s(t). \qquad (3)$$

The solution to equation (3) can be written in term of the Green's function as

$$\begin{aligned}\delta p(z,t;z_s)&=\int G(z',t;z,0)*\left[\delta(z'-z_s)\delta s(t)\right]dz'\\&=G(z_s,t;z,0)*\delta s(t)\\&=\int G(z_s,t'-t;z,0)\delta s(t')dt'\\&=\int G(z_s,t';z,t)\delta s(t')dt'\end{aligned} \qquad (4)$$

The pressure field is recorded at $z=z_g$. Thus, we have

$$V(z_g,t;z_s\,|\,t')=\frac{\partial p(z_g,t;z_s)}{\partial s(t')}=G(z_g,t;z_s,t') \qquad (5)$$

and

$$\delta s(t')=V^T(t'|\,z_g,t;z_s)\delta p(z_g,t;z_s)=V(z_g,t;z_s\,|\,t')\delta p(z_g,t;z_s). \qquad (6)$$

Therefore, the perturbation in the source function can be written as

$$
\begin{aligned}
\delta s(t') &= \sum_s \sum_g \int G(z_g, t; z_s, t') \delta p(z_g, t; z_s) dt \\
&= \sum_s \sum_g G(z_g, -t'; z_s, 0) * \delta p(z_g, t'; z_s).
\end{aligned} \tag{7}
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