Application of Hybrid Linear and Non-linear Full-Waveform Inversion to Gulf of Mexico Data

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SUMMARY

Full-Waveform Inversion (FWI) of seismic data often suffers from poor sensitivity to deep features of the subsurface model. This degrades the quality of seismic images at deep portions. We propose a hybrid linear and non-linear optimization method to improve the accuracy of FWI tomogram updating along deep reflection wavepaths, which is often below the reach of diving-waves. The algorithm is implemented in the space-time domain to simultaneously invert a range of frequencies. We apply the method to a marine Gulf of Mexico field data and illustrate the success of the method after several iterations.

INTRODUCTION

For deep subsurface imaging, waveform inversion (Tarantola, 1984) should invert deeper reflections rather than first arrivals. Unfortunately, standard FWI has low sensitivity to waveform residuals related to deeper reflections compared to the stronger amplitude diving waves. The consequence is slow and often inadequate FWI convergence for reconstructing deep portions of the velocity model. To enhance the sensitivity of FWI to deeper reflections, we use a linear-inversion scheme instead of reverse-time-migration (RTM) (Baysal et al., 1983) for calculating velocity updates. Using this linear inversion, sharp boundaries are incorporated into the velocity model such that they implicitly enhance the velocity updating along the reflection wavepaths. This linear inversion is least-squares reverse-time migration (LSRTM) (Dai et al., 2012).

Using LSRTM, a reflectivity model is computed based on the Born approximation, where the background velocity is fixed during the linear inversion. The velocity model is then updated with the reflectivity model. After that, the linear inversion is repeated with the updated velocity model as a background velocity. Each linear-inversion and updating of the velocity model constitute a non-linear iteration. This combined linear and non-linear inversion procedure is cyclically repeated until acceptable convergence.

To avoid high computational and memory costs, the linear inversion is computed by an iterative Conjugate Gradient solver. This optimization is generally similar to Gauss-Newton-Krylov FWI (Akcelik et al., 2002; Erlangga and Herrmann, 2009). We implemented the algorithm in the time-domain to invert the data for a band of frequencies, starting from a narrow band of low frequencies and progressively include higher frequencies into the inversion.

The algorithm for the Hybrid FWI is first reviewed and then applied to Gulf of Mexico (GOM) data. The results illustrate the success of the method in enhancing reflection wavepaths and producing high quality migration images.

THEORY

Newton's method for minimizing the difference $\delta\mathbf{d}$ between the calculated and observed data can be written as

$$\mathbf{s}_{k+1} = \mathbf{s}_k - \mathbf{H}_f^{-1}(\mathbf{s}_k)\nabla f(\mathbf{s}_k), \tag{1}$$

where $\mathbf{s}_k$ is the slowness model, $\mathbf{H}_f$ is the Hessian matrix and $\nabla f(\mathbf{s}_k)$ is the gradient of the objective function $f(\mathbf{s}_k) = \frac{1}{2}\|\delta\mathbf{d}(\mathbf{s}_k)\|_2^2$ at the $k-$th iteration. By approximating the Hessian as $\mathbf{H} \approx (\mathbf{J^T J})$, where $\mathbf{J}$ is the Jacobian matrix. A line search is used to estimate the step length α_k because the approximation of the Hessian might not be an accurate estimate of the curvature for the non-linear misfit function. Instead of inverting the Hessian matrix, we iteratively solve the system of equations

$$\left(\mathbf{J^T J}\right)\mathbf{g} = \mathbf{J}^T\delta\mathbf{d}_k, \tag{2}$$

using the same velocity model to get the search direction $\mathbf{g}$. In other words, LSRTM is used to compute the search direction $\mathbf{g}$ instead of the RTM. Once the search direct direction $\mathbf{g}$ and the line-search parameter α are found, the velocity model is updated using

$$\mathbf{s}_{k+1} = \mathbf{s}_k - \alpha_k\mathbf{g}, \tag{3}$$

and the Jacobian operator updated according to the new velocity model.

APPLICATION TO GOM STREAMER DATASET

We apply the Hybrid FWI to streamer data from the Gulf of Mexico. There are 515 shots with a 37.5 meter interval, and the source-receiver offsets are from 198 meters to 6 kilometers, with a 12.5 meter receiver spacing. The trace length is 10 seconds with a 2 ms sampling interval. Prior to inversion, the data spectra are multiplied by $\sqrt{i/\omega}$ and gained by $\sqrt{t}$ in the time domain to transform 3D to 2D geometric spreading. The source wavelet is estimated by stacking early arrivals from the near-offset traces.

We start the inversion with the data bandpassed filtered from 0-4 Hz, where there is reliable signal around 4 Hz. The grid size for the tomograms is 300 by 1600 grid points in the vertical and horizontal directions, respectively, with a grid point spacing of 12.5 meters. Figures 1a and b show the initial and final tomograms after 70 iterations.

The inversion is stopped every few non-linear iterations (about 5 iterations) depending on the convergence rate. We start the inversion again from where we stop with the bandwidth of the observed data widened, and the velocity of the water layer is set to 1500 m/s.

Figure 1c shows the wavepaths computed by migrating the shot gather with the shot positioned in the middle of the survey. The strong, curved wavepaths are related to diving waves. Those wavepaths do not reach depths below 1.5 km, which indicate that the shallow section above 2 km is constrained mainly by the diving waves. The arrow in the figure indicate a reflection wavepath associated with the deeper reflections which are emphasized by the inversion. Those wavepaths implicitly constrain the model below the reach of diving waves.

Figure 1d shows the Kirchhoff migration image using the Hybrid FWI tomogram, and 1d shows the common image gathers (CIG's) which have mostly flat events indicating a good velocity model in both the shallow and deep sections.

CONCLUSION

We implemented and applied time-domain Hybrid linear and non-linear FWI to a GOM dataset. The algorithm uses LSRTM images as velocity updates instead of RTM images. In the deep part of the section, the Hybrid FWI uses the deep reflection data to define sharp boundaries in the velocity model. These sharp boundaries generate wavepaths that are used by the inversion to build velocity updates for the deeper section.

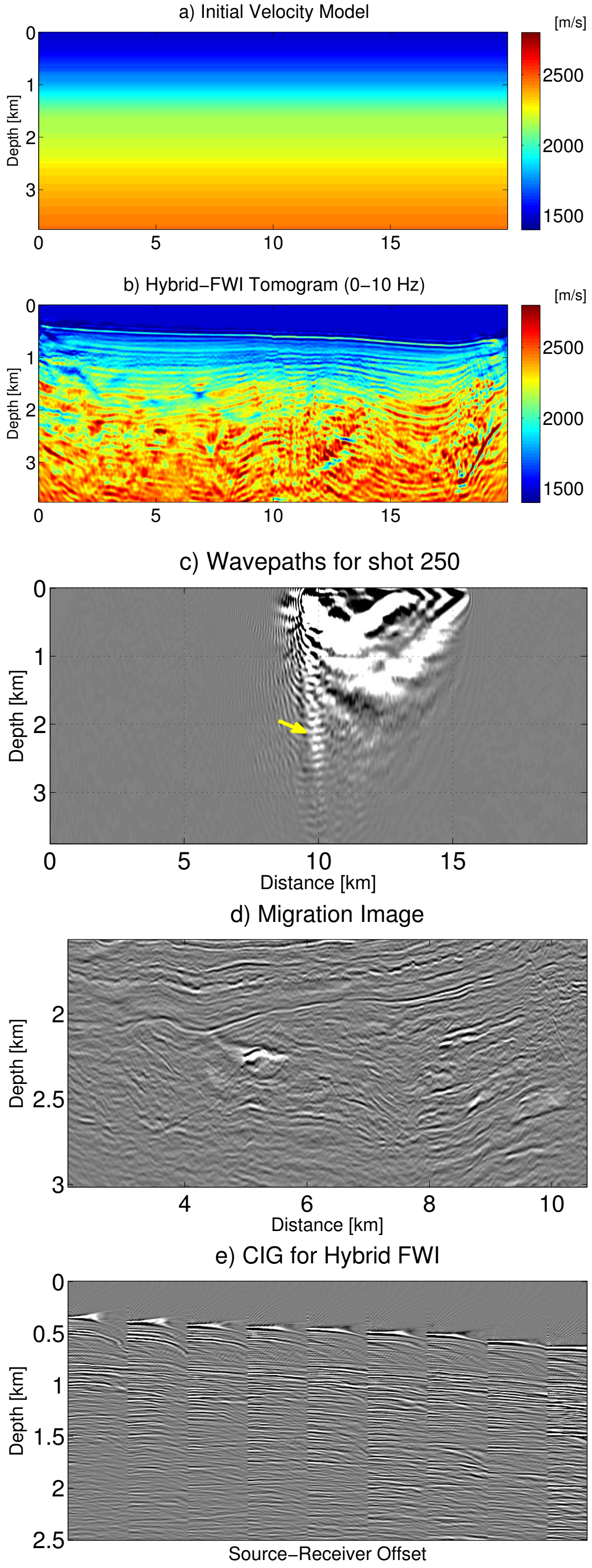

Figure 1: a) the initial velocity model, b) final Hybrid FWI tomogram of the band 0-10 Hz, c) migration images of a single shot gather showing wavepaths, d) the migration image and c) common image gathers.

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