# Multiples Waveform Inversion

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## Abstract

To increase the illumination of the subsurface and to eliminate the dependency of FWI on the source wavelet, we propose multiples waveform inversion (MWI) that transforms each hydrophone into a virtual point source with a time history equal to that of the recorded data. These virtual sources are used to numerically generate downgoing wavefields that are correlated with the backprojected surface-related multiples to give the migration image. Since the recorded data are treated as the virtual sources, knowledge of the source wavelet is not required, and the subsurface illumination is greatly enhanced because the entire free surface acts as an extended source compared to the radiation pattern of a traditional point source. Numerical tests on the Marmousi model show that the convergence rate and the spatial resolution of MWI is, respectively, faster and more accurate then FWI. The potential pitfall with this method is that the multiples undergo more than one roundtrip to the surface, which increases attenuation and reduces spatial resolution. This can lead to less resolved tomograms compared to conventional FWI. The possible solution is to combine both FWI and MWI in inverting for the subsurface velocity distribution.

## Theory

### Algorithm of MWI

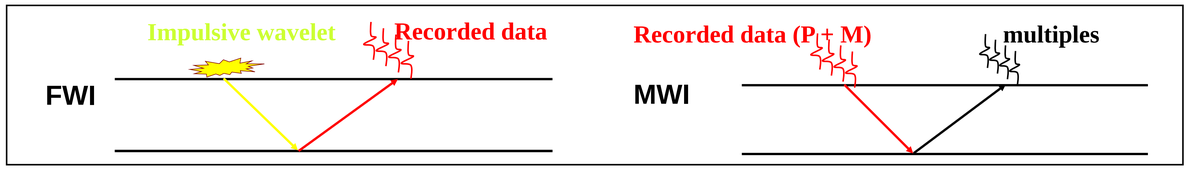
1. $\varepsilon = 1/2 \sum_{\omega} \sum_{g} \sum_{s} \Delta M(\mathbf{x}_g, \mathbf{x}_s)^* \Delta M(\mathbf{x}_g, \mathbf{x}_s)$

2. $g(\mathbf{x}) = \mathrm{Real}[2s(\mathbf{x}) \sum_{\omega} \sum_{g} \sum_{s} \omega^2 F(\mathbf{x})^* B(\mathbf{x})]$

3. $s(\mathbf{x})^{i+1} = s(\mathbf{x})^i - \alpha g(\mathbf{x})$

### MWI Versus FWI

| | Source wavefield | Receiver wavefield |
|---|---|---|
| FWI | Impulsive wavelet | Recorded data |
| MWI | Recorded data (P+M) | Multiples |




### Generate Multiples

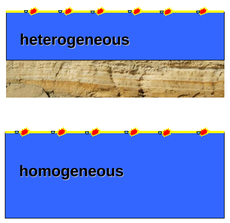
**Step 1**

Line source (P +M) → direct propagation → $P^d+M^d$

→ reflected propagation → $M^r$

**Step 2**

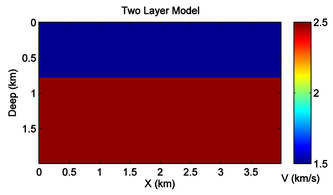
Line source (P +M) → direct propagation → $P^d+M^d$

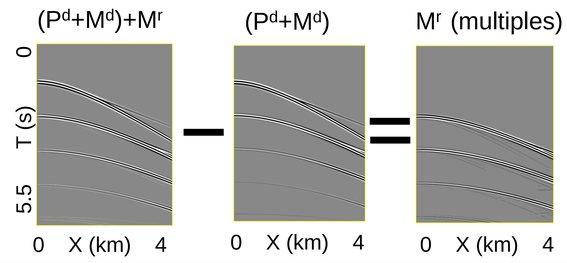



**Step 3**

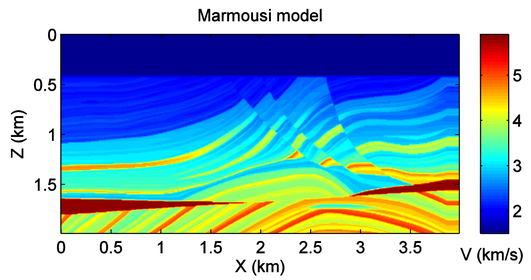
$M^r = \overbrace{(P^d+M^d)+M^r}^{\text{heterogeneous}} - \overbrace{(P^d+M^d)}^{\text{homogeneous}}$

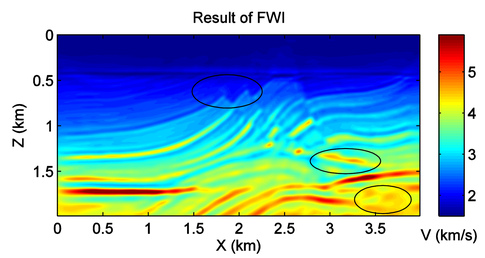
**Example**

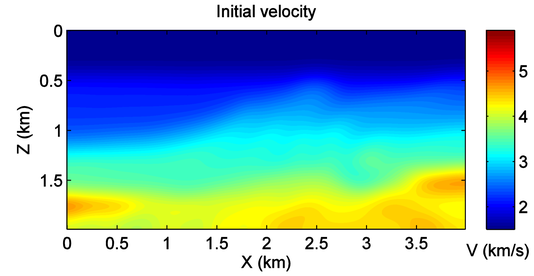


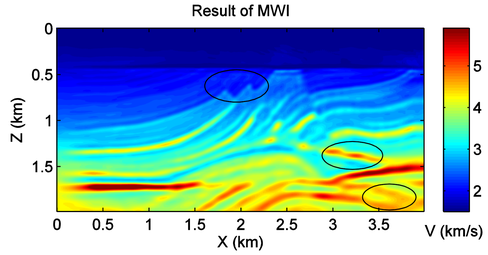



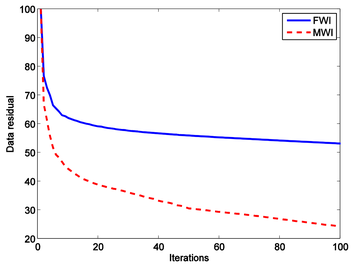

## Results: Marmousi Model

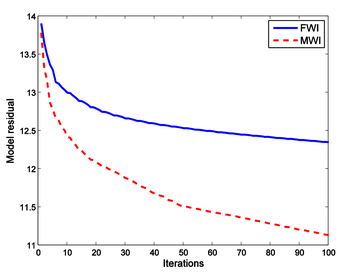


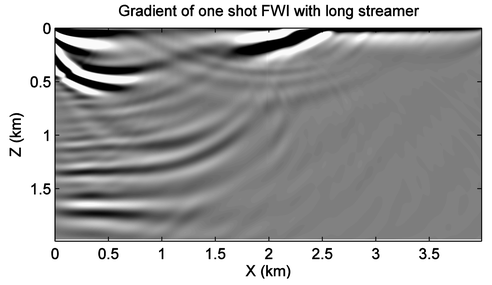



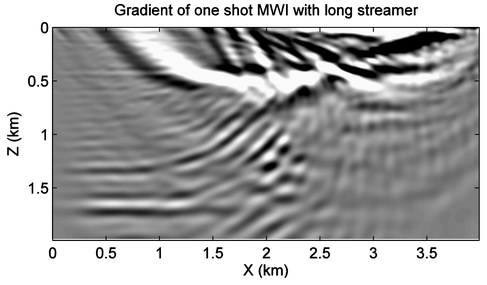




## Conclusion

We propose multiples waveform inversion to invert the surface-related multiples for the subsurface velocity distribution. In this method, recorded traces are used as the time histories of the virtual sources at the hydrophones and surface-related multiples are the observed data. Numerical tests on the Marmousi model verify that MWI is a promising new method for velocity inversion. Compared to standard FWI, the advantages of MWI are that it uses the recorded trace as a source wavelet so no knowledge of the actual source wavelet is needed, and it enjoys faster convergence and higher resolution compared to FWI with the Marmousi data.