# **Multisource Early-Arrival Waveform Inversion of Crosswell Data**

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## Summary

We present an efficient and robust multisource early-arrival waveform inversion (EWI) method using dynamic muting windows to overcome the problems of high computational cost and slow convergence in conventional waveform inversion of VSP or crosswell data. Numerical results on both synthetic and field data show very stable convergence and a 12X speedup in computational efficiency for the Friendswood data without any significant loss in accuracy of the inverted tomograms. The method proposed here can be used for robustly and efficiently estimating statics for exploration seismology and deep earth structure for earthquake seismology.

#### Introduction

Full Waveform Inversion (FWI) can provide higher resolution estimates of the velocity model compared to traveltime tomography. However, its implementation is very expensive due to many iterations of forward modeling and back-projection of the residual wavefields. For real data, the implementation becomes difficult because of the non-linearity of the problem and insufficient physics (attenuation, elastic effects, anisotropy etc.).

We overcome the high computational cost of FWI by the use of multisource phase-encoded waveform inversion (Krebs et al. (2009), Ben-Hadj-Ali et al. (2011)). To achieve robust convergence, we use dynamic muting windows where we use the first-arrival traveltimes for each common shot gather (CSG), and then superimpose a narrow window about the first arrivals. The windowed arrivals are then phase-encoded and blended together to form a supergather, which is then inverted. The windowed mask is also blended together to form a supergather mask that is imposed upon the predicted supergather. The window length is then gradually increased until all the reflections arrivals are admitted into the inversion. This is a robust and efficient method for inversion of VSP/crosswell data and provides a viable alternative to the multiscale approach of Bunks et al. (1995) or Boonyasiriwat et al. (2009).

### Theory of FWI and Multisource FWI

FWI seeks to reconstruct the earth's model parameters such as velocity and density from the recorded waveform data. In conventional acoustic FWI, the velocity model, V(**x**), where  $\mathbf{x}=\{x,y,z\}$ , is obtained by matching the predicted data,  $P_{\text{cale}}(\mathbf{x}_{\text{s}},\mathbf{x}_{\text{g}},\omega)$ , with the observed data,  $P_{\text{obs}}(\mathbf{x}_{\text{s}},\mathbf{x}_{\text{g}},\omega)$ , where *P* denotes the pressure field,  $\mathbf{x}_{\text{s}}$  and  $\mathbf{x}_{\text{g}}$  denote the source and receiver locations, respectively. According to Tarantola (1984), the misfit function,  $\varepsilon$ , can be formulated as,

$$\varepsilon = \frac{1}{2} \sum_{\omega} \sum_{s} \sum_{g} \left\| \Delta P(\mathbf{x}_{s}, \mathbf{x}_{g}, \omega) \right\|^{2},$$

where  $\Delta P(\mathbf{x}_s, \mathbf{x}_g, \omega) = P_{calc}(\mathbf{x}_s, \mathbf{x}_g, \omega) - P_{obs}(\mathbf{x}_s, \mathbf{x}_g, \omega)$ , and the summation is over source frequencies and source-geophone coordinates. The slowness model,  $s(\mathbf{x})^i$ , at the *i*-th iteration is updated using a gradient descent algorithm,

$$s(\mathbf{x})^{(i+1)} = s(\mathbf{x})^{(i)} - \alpha \gamma(\mathbf{x})^{(i)},$$

where  $\alpha$  is the step length and  $\gamma(\mathbf{x})$  is the misfit gradient. The gradient can be written as the reverse time migration (RTM) of data residuals (Tarantola (1984)) as,

$$\gamma(\mathbf{x}) = \sum_{\omega} S(\mathbf{x}, \omega) R^*(\mathbf{x}, \omega),$$

where  $S(\mathbf{x},\omega)$  and  $R(\mathbf{x},\omega)$  represent the source and residual wavefields, respectively. For multisource FWI, the gradient gets modified as,

$$\widehat{S}(\mathbf{x},\omega) = \sum_{j=1}^{N} a_{j}(\omega) S_{j}(\mathbf{x},\omega), \text{ and, } \widehat{R}(\mathbf{x},\omega) = \sum_{j=1}^{N} a_{j}(\omega) R_{j}(\mathbf{x},\omega),$$

where  $a(\omega)$  is the phase-encoding function and N is the number of CSGs phase-encoded together. The gradient of multisource FWI thus gets modified as,

$$\widehat{\gamma}(\mathbf{x}) = \sum_{\omega} \widehat{S}(\mathbf{x}, \omega) \widehat{R}^*(\mathbf{x}, \omega) = \sum_{j=1}^{N} \sum_{\omega} |a_j(\omega)|^2 S_j(\mathbf{x}, \omega) R_j^*(\mathbf{x}, \omega) + \sum_{j \neq k}^{N} \sum_{k=1}^{N} a_j(\omega) a_k^*(\omega) S_j(\mathbf{x}, \omega) R_k^*(\mathbf{x}, \omega).$$

For multisource EWI, we use the first-arrival traveltimes for each shot gather and create a window mask about the early-arrivals. CSG masks from different shot gathers are then phase-encoded and blended together to form a supergather mask that mostly admits only the first-arrivals in the supergather. A supergather formed using this method is shown in Figure 1. A combination of source-time statics and polarity encoding, suggested by Romero et al. (2000), are used as encoding functions that are dynamically changed at every iteration. The same supergather masks are also imposed upon the predicted supergathers. This is an efficient strategy for avoiding the local minima problem in multisource waveform inversion. This encoding strategy also gives the freedom of choosing a large variance of the source-time static shifts that is necessary to avoid crosstalk noise in the gradient.

### Examples

We first demonstrate the application of our proposed method on a synthetic model shown in Figure 2(a). The acquisition geometry for a crosswell survey is chosen where the sources and receivers placed in a vertical well on

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the left and right sides of the model, respectively. Figures 2(b) and 2(c) compare the tomograms from standard and multisource FWI, respectively. For the multisource inversion, all the CSGs are phase-encoded and blended together into one supergather. It is evident from Figures 2(b) and 2(c) that the multisource waveform tomogram is almost as accurate as the standard waveform tomogram.



Figure 1: Phase-encoding of windowed early-arrival CSGs to form a supergather for multisource EWI.



Figure 2: (a) The true velocity model, (b) standard FWI tomogram, (c) multisource waveform tomogram with 200 shots phaseencoded into one supergather.

For the field data example, we use the Friendswood crosswell data (Chen at al. (1990)). There were 98 sources and 96 receivers in the source and receiver wells respectively. The processing steps used for the field data are: (1) phase and amplitude correction from 3D to 2D, (2) directional 9-point median filter to eliminate the tube waves, (3) bandpass filter to remove any extreme noise in the data. The first-arrival traveltime tomogram, shown in Figure 3(a), is used as the starting model for conventional and multisource EWI using the phase-encoding strategy suggested in the previous section. Figures 3(b) and 3(c) show the standard and multisource EWI tomograms, respectively, where all the 98 shots have been phaseencoded together into one supergather for the multisource inversion. To improve the signal-to-noise (SNR) ratio of the gradient, we iteratively stacked 8 gradients with different encoding functions to obtain the average gradient at every iteration. Hence, the speedup in computational efficiency per iteration of the inversion is  $98/8 \approx 12X$ . Figures 4(a) and 4(b) compare the RTM images using the velocity tomograms in Figures 3(b) and 3(c), respectively. The RTM images in both cases are very similar. Another accuracy check is the comparison shown in Figure 4(c)between a smoothed sonic  $\log (12 m \text{ from the source well})$ and a vertical slice of the multisource tomogram at the same location. The multisource waveform tomogram provides a reasonable fit to the sonic log that further establishes its accuracy.

#### Conclusions

We presented an efficient and robust multisource EWI method for computing velocity tomograms and applied it on synthetic and field crosswell data. For robust convergence, we used only the first arrivals in the inversion initially and as the inversion progressed, we admitted the later arrivals into the inversion. The fidelity of the multisource EWI tomogram is validated by examining the RTM images and the sonic log. A computational speedup of 12X per iteration of the inversion is achieved without any significant compromise in the image quality. Such an order of reduction in computational cost becomes a huge factor especially when dealing with 3D data.

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Figure 3: (a) Traveltime tomogram, (b) standard EWI tomogram, (c) multisource EWI tomogram with one supergather.



Figure 4: RTM images from (a) conventional, (b) multisource EWI tomograms, (c) comparison with well log.

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