Wave Equation Traveltime Inversion

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Outline

- Theory
- Implementation
- Results
- Summary

Theory

Objective Function

$$\epsilon = \frac{1}{2}\sum_{s,g} \Delta\tau_{s,g}^2$$

$\Delta\tau_{s,g}$ is the time lag that maximizes the cross-correlation

$$f(\tau, s) = \int_{-\infty}^{+\infty} P_{cal}(t)\, P_{obs}(t+\tau)dt$$

At the maximum point, the derivative is zero, and doesn't change, so by implicit function theory, we have:

$$\frac{\partial\Delta\tau}{\partial s} = -\frac{\dfrac{\partial \frac{\partial f}{\partial\tau}}{\partial s}}{\dfrac{\partial^2 f}{\partial\tau^2}} = -\frac{\int \frac{\partial P_{cal}}{\partial s}(t)\,\frac{\partial P_{obs}(t+\tau)}{\partial\tau}dt}{\int P_{cal(t)}\,\frac{\partial^2 P_{obs}(t+\tau)}{\partial\tau^2}dt}$$

Theory

We want to know how the objective function change if we change the model:

$$\frac{\partial \epsilon}{\partial s} = \sum_{s,g} \Delta\tau \frac{\partial \Delta\tau}{\partial s}$$

Combine the two equation we get:

$$\frac{\partial \epsilon}{\partial s} = \sum_{s,g} -\Delta\tau \frac{\int \frac{\partial P_{cal}}{\partial s}(t) \frac{\partial P_{obs}(t+\tau)}{\partial \tau} dt}{\int P_{cal(t)} \frac{\partial^2 P_{obs}(t+\tau)}{\partial \tau^2} dt}$$

$$= \sum_{s,g} \int \frac{\partial P_{cal}}{\partial s}(t) \frac{-\Delta\tau \frac{\partial P_{obs}(t+\tau)}{\partial \tau}}{\int P_{cal(\theta)} \frac{\partial^2 P_{obs}(\theta+\tau)}{\partial \tau^2} d\theta |_{\tau=\Delta\tau}} dt$$

Theory

Recall the gradient of objective function in FWI:

$$\frac{\partial \epsilon}{\partial s} = \sum_{s,g} \int \frac{\partial P_{cal}}{\partial s}(t)(P_{cal}(t) - P_{obs}(t))\ dt$$

To get a WT program, we only need to change

1. Data residual $P_{cal}(t) - P_{obs}(t)$ to $\frac{-\Delta\tau \frac{\partial P_{obs}(t+\tau)}{\partial \tau}}{\int P_{cal(\theta)} \frac{\partial^2 P_{obs}(\theta+\tau)}{\partial \tau^2} d\theta |_{\tau=\Delta\tau}}$
2. Objective function.

Implementation

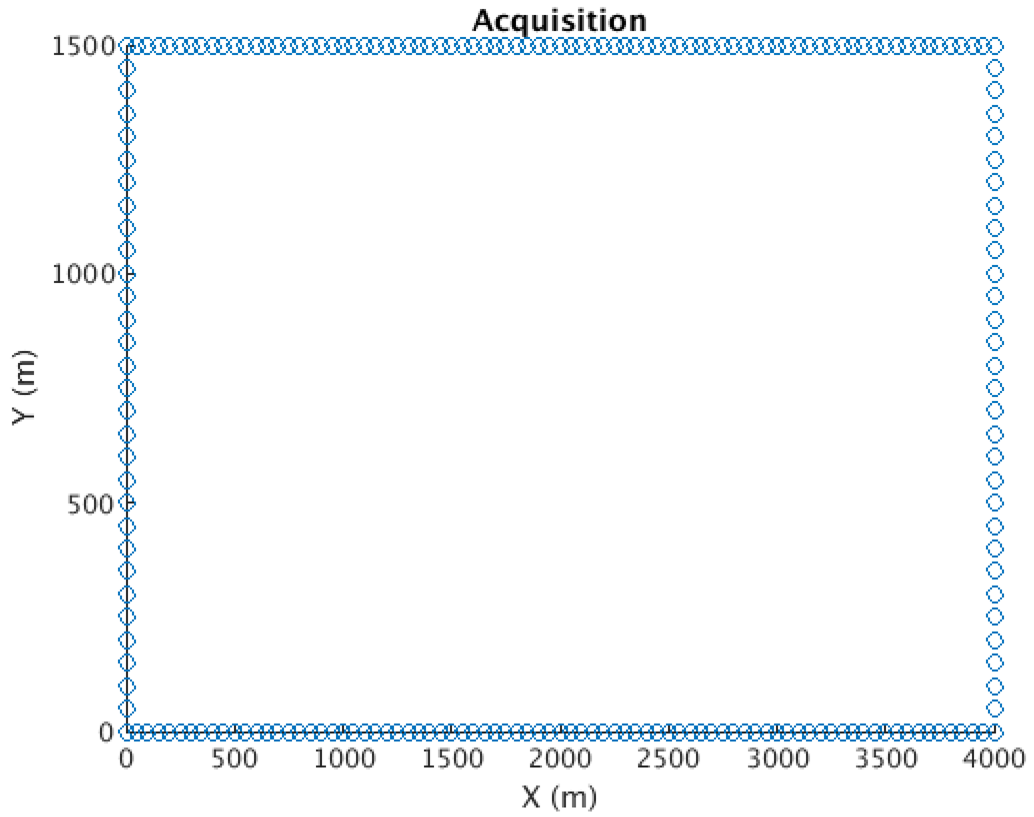
- Understand a FWI program (Xin Wang's)
- Calculate $\frac{-\Delta\tau \frac{\partial P_{obs}(t+\tau)}{\partial\tau}}{\int P_{cal(\theta)} \frac{\partial^2 P_{obs}(\theta+\tau)}{\partial\tau^2} d\theta|_{\tau=\Delta\tau}}$

1. Calculate $\Delta\tau$
2. Shift the observed data by $\Delta\tau$, take the derivative
3. Calculate the denominator

- Change the objective function to

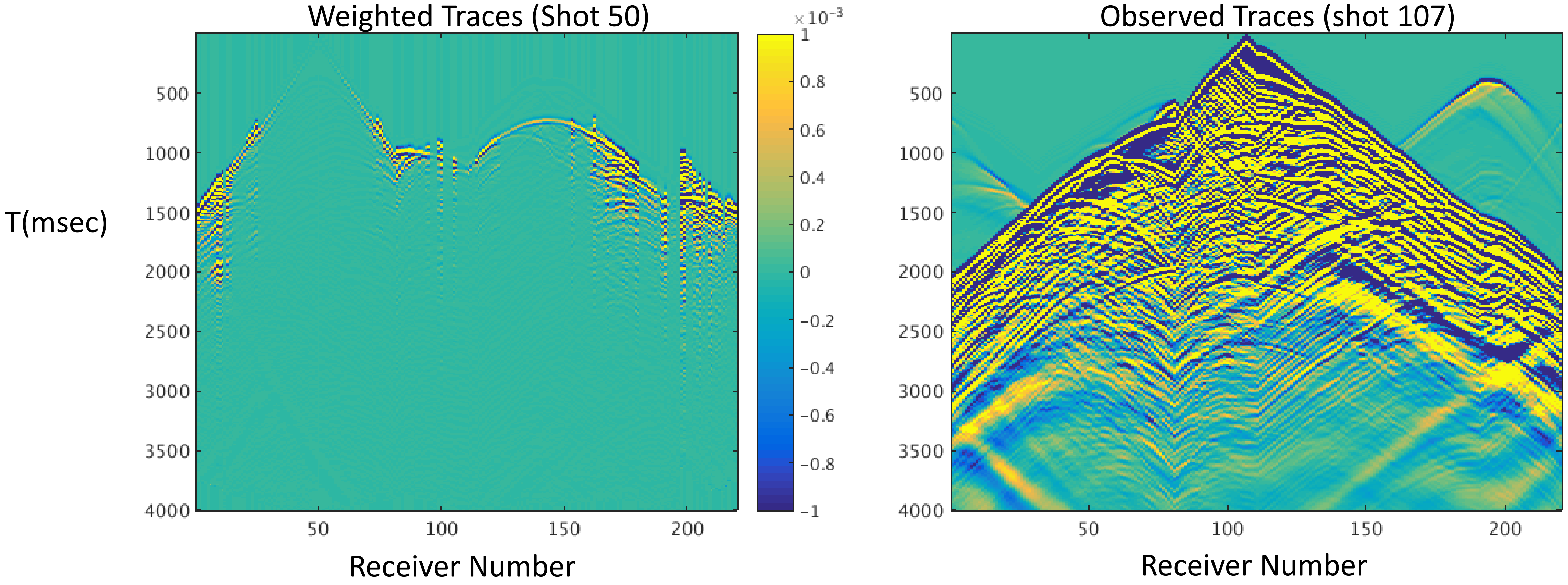
$$\epsilon = \frac{1}{2}\sum_{s,g} \Delta\tau_{s,g}^2$$

Implementation



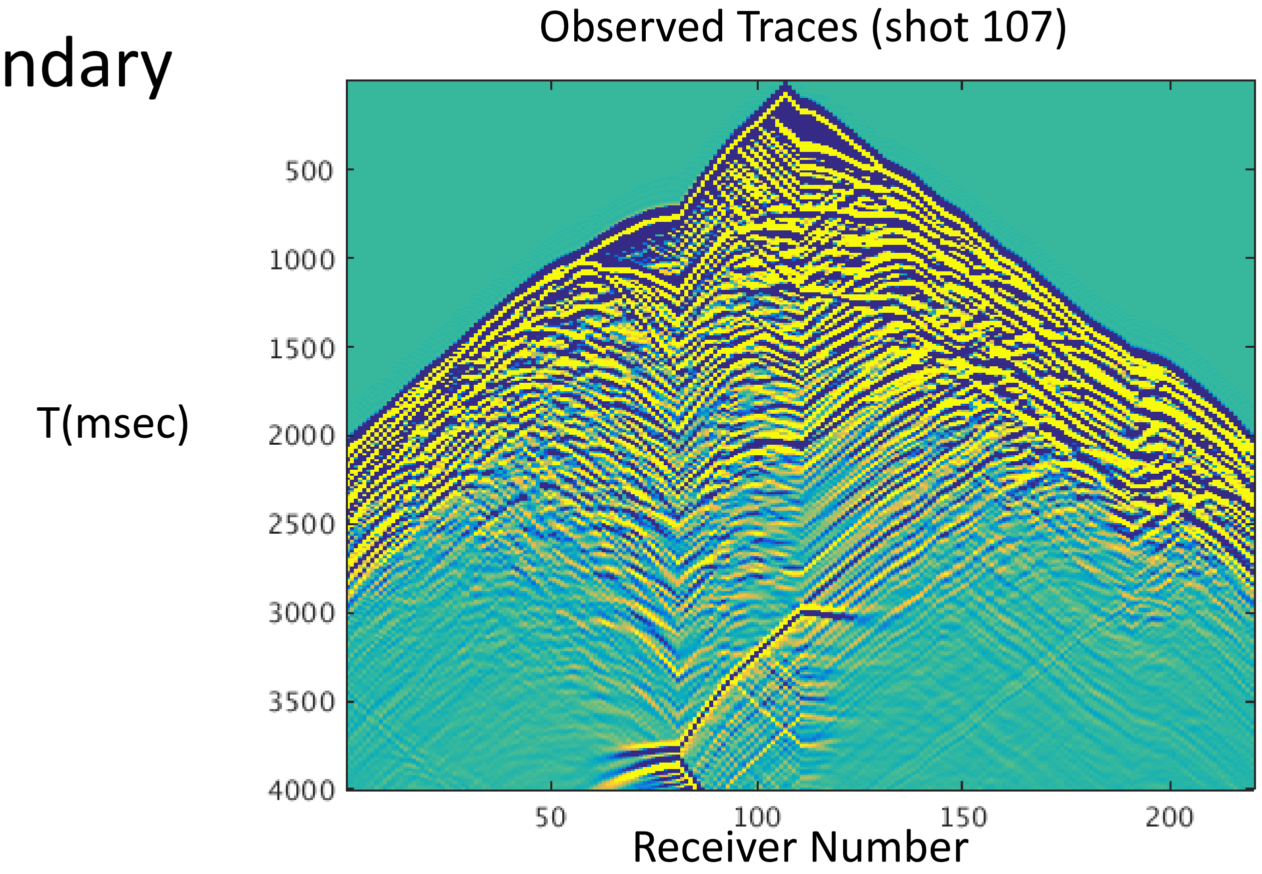
Implementation

- For complex model, calculation of weighted traces can be a problem.

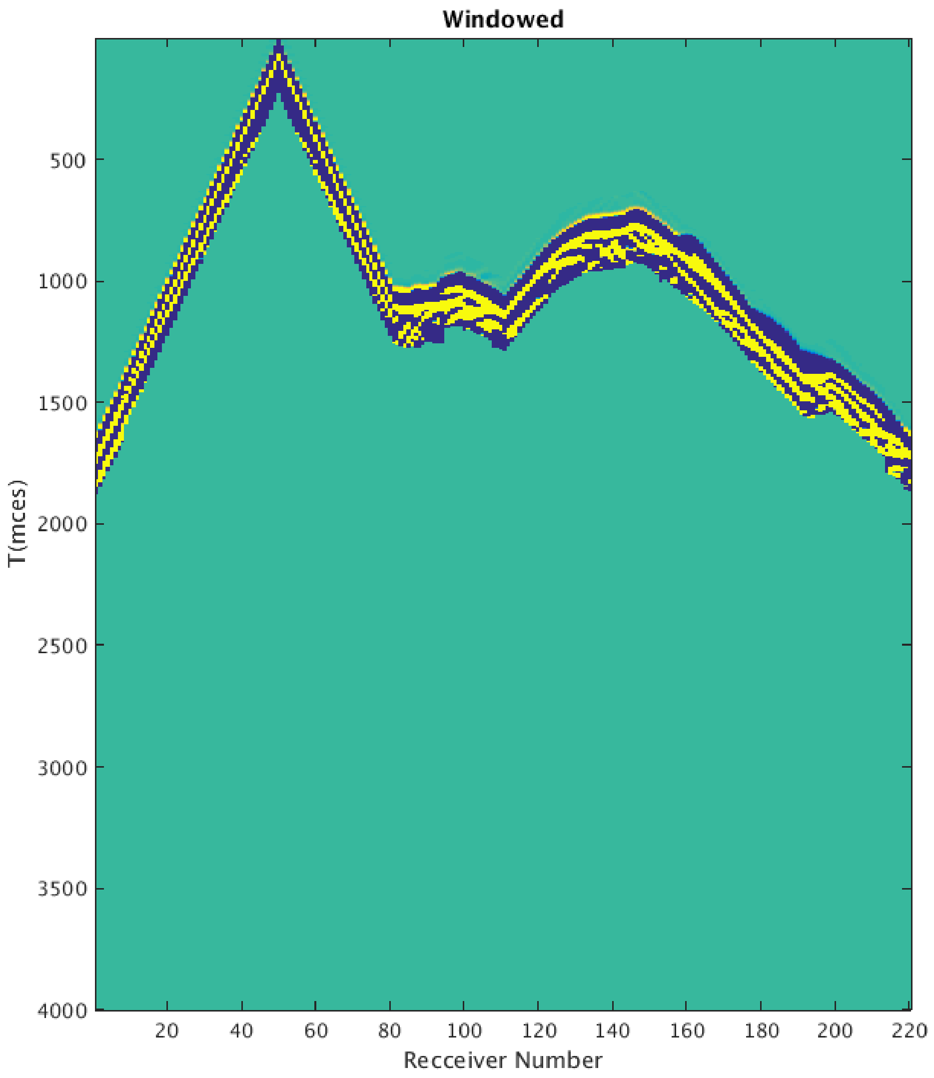


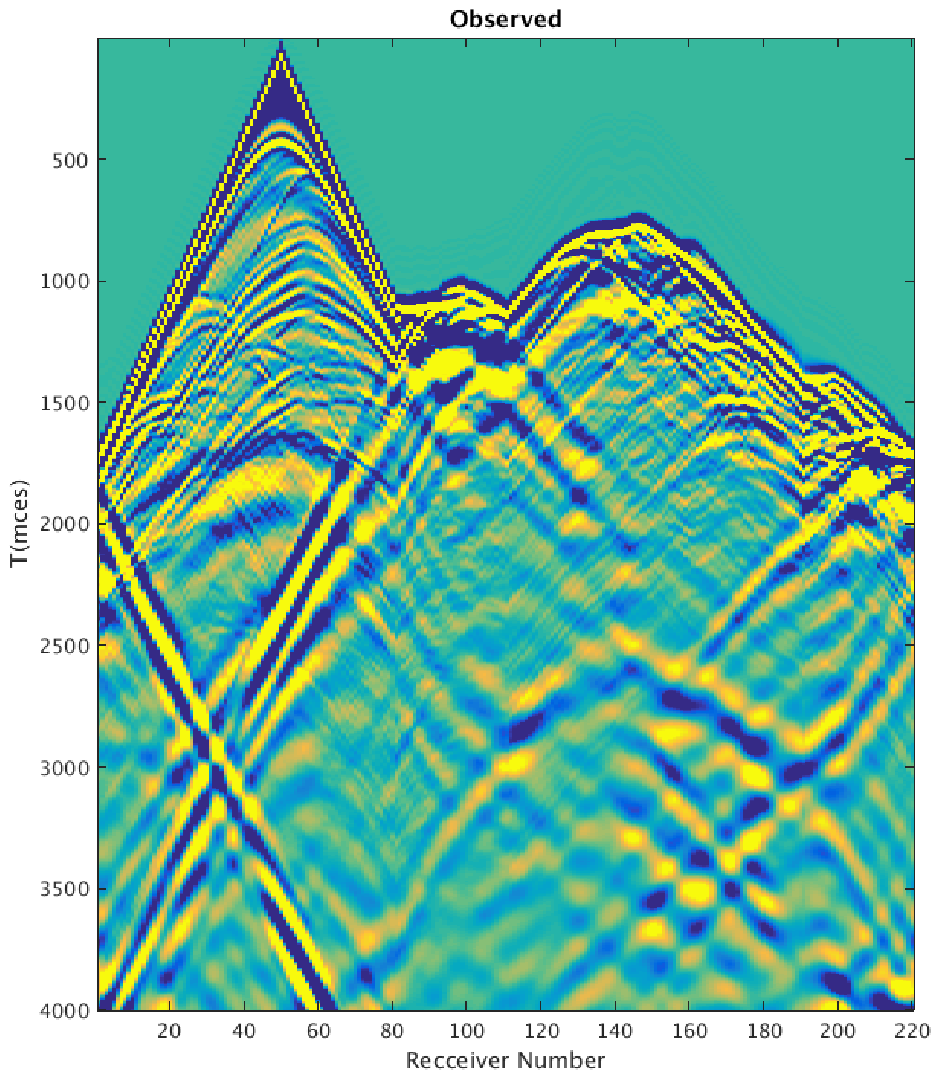
Implementation

Better result for bigger boundary

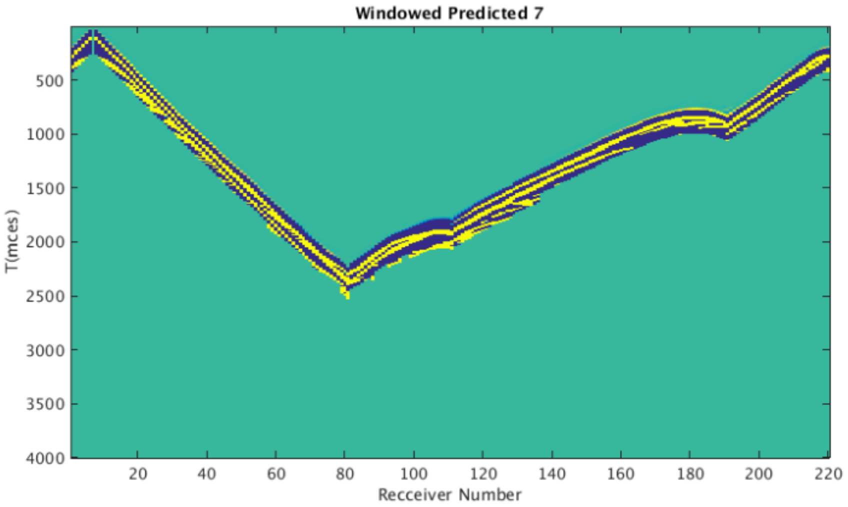


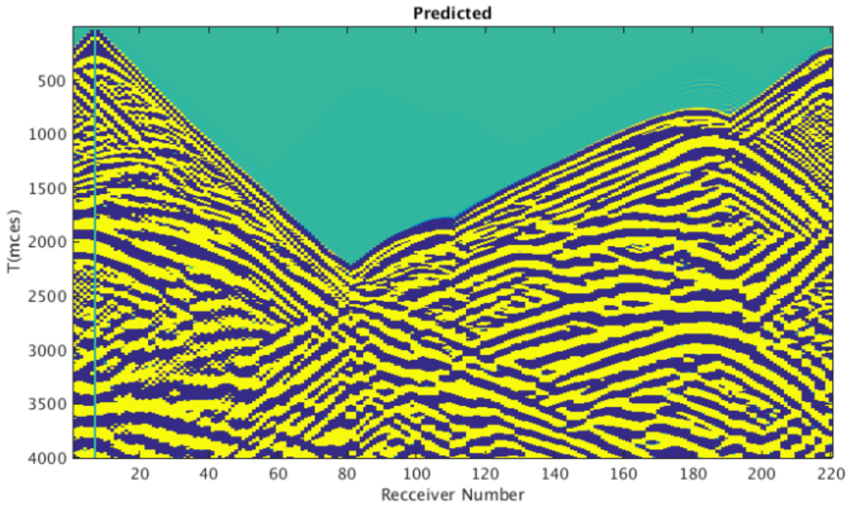
Implementation

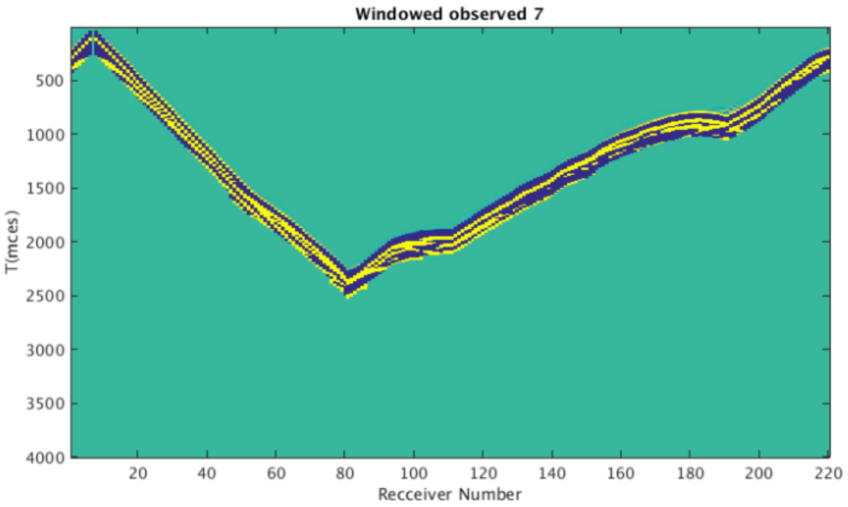


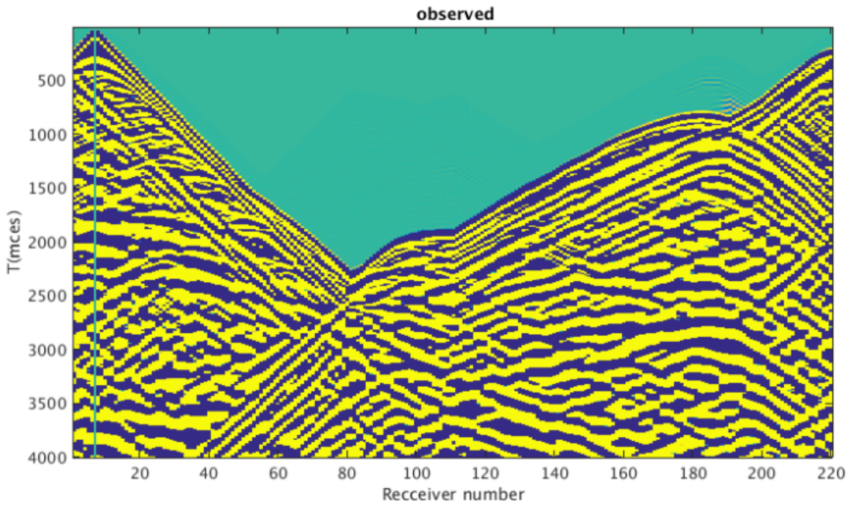


Implementation

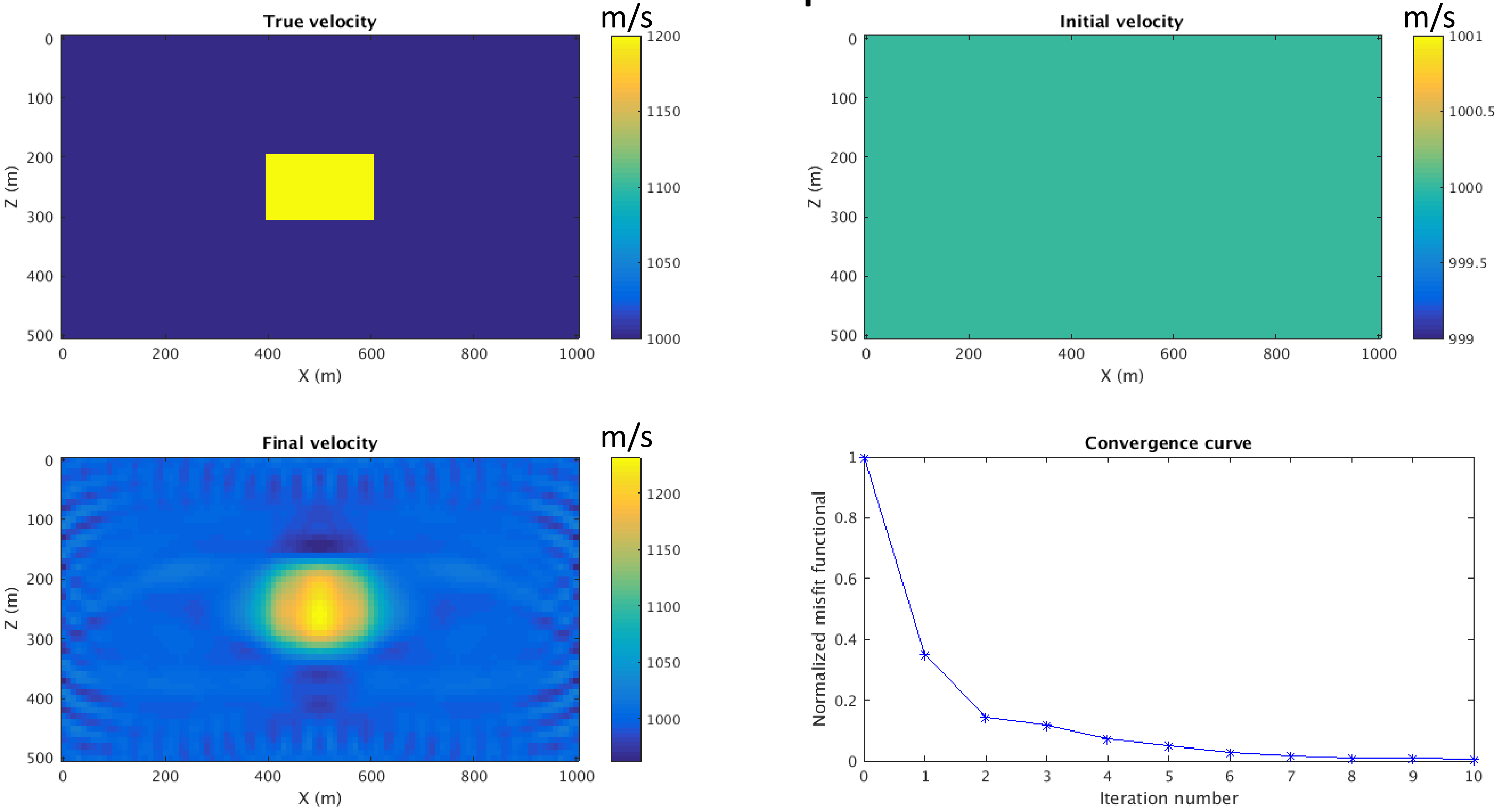




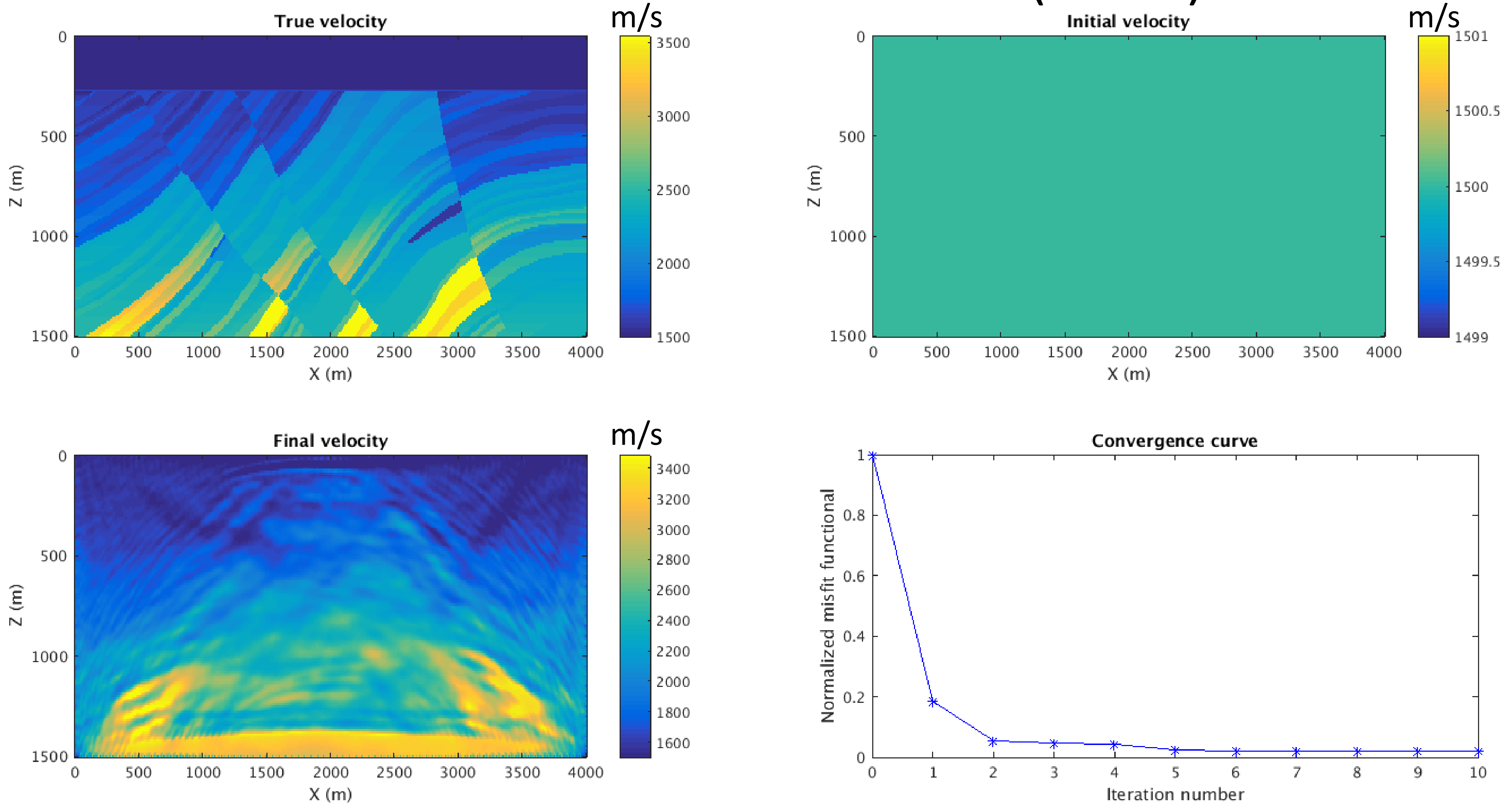




Results of Simple Model



Results of Marmousi2 (Part)



Summary

- FWI-like workflow
- Essentially tomography
- Limited resolution (even for closed acquisition)
- $\Delta\tau$ calculation

1. Estimate the first arrival(need for field data)
2. Mute all other sigals
3. Cross-correlation